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Evolutionary Stability of Bargaining and Price Posting: Implications for Formal and Informal Activities*

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Abstract

In this paper we study the co-existence of two well known trading protocols, bargaining and price-posting. To do so we consider a frictional environment where buyers and sellers play price-posting and bargaining games infinitely many times. Sellers switch from one market to the other at a rate that is proportional to their payoff differentials. Given the different informational requirements associated with these two trading mechanisms, we examine their possible co-existence in the context of informal and formal markets. Other than having different trading protocols, we also consider other distinguishing features. We find a unique stable equilibrium where price-posting (formal markets) and bargaining (informal markets) co-exist. In a richer environment where both sellers and buyers can move across markets, we show that there exists a unique stable dynamic equilibrium where formal and informal activities also co-exist whenever sellers' and buyers' net costs of trading in the formal market have opposite signs.

JEL Codes: C7, D49.

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1 Introduction

The literature examining the emergence and persistence of various trading protocols has a long tradition in economics. Among trading protocols, bargaining was the predominant selling institution, while auctions appeared later, as shown by Cason et al. (2003), Milgrom and Weber (1982), and Krishna (2003). In contrast, price-posting by sellers is a relatively recent phenomenon which can be dated back to 1823, when Alexander Stewart introduced posted prices in his New York City 'Marble Dry Goods Palace'.

Trying to understand which trading mechanisms survive is the focus of some of the evolutionary literature. In a seminal article, Lu and McAfee (1996) consider a frictional environment where a large number of homogeneous buyers and sellers trade in markets. Agents who transact leave the market, and a fraction of the remaining agents are also removed. All these agents are replaced by new identical players. Within this framework, the authors show that both auctions and bargaining are equilibria. But only auctions are evolutionarily stable. In the same spirit, Kultti (1999) analyzes posted price markets and auctions. The author finds that posted price markets are equivalent to those that use auctions.[1] This is because in auctions, the seller settles for his reservation utility if only one buyer appears, whereas if many buyers visit the seller, it is the buyers who must settle for their reservation utility. Instead in posted price, the seller's share of the surplus is the same regardless of the number of buyers who visit him, as shown by Burdett, Shi and Wright (2001). Here, we complement the previous analysis by studying bargaining and price posting.[2]

In this paper, we explore the potential co-existence of price posting and bargaining markets. We do so in the context of formal and informal activities. This is a natural avenue to explore such issues, as the informational demands of price posting and bargaining are very different, in ways that correspond to formal and informal activities, respectively. Following the evolutionary setup of Lu and McAfee (1996), in this paper we determine under what circumstances bargaining (informal markets) and price posting (formal markets) can co-exist and deliver a unique stable equilibrium.[3] In doing so, we also contribute to the literature that studies informal activities. As opposed to Lu and McAfee (1996) and Kultti (1999), here every agent leaves the market at the end of each period and their offspring replace them.[4] We then consider best response dynamics,

[1]With homogenous reservation prices and small markets, however, Julien, Kennes, and King (2001) show that auctions are preferred by sellers.

[2]Lu and McAfee (1996) point out "in markets where bargaining persists, there must be some factors which make bargaining preferable to auctions. The effects of such factors must be strong enough to override the structural advantages of auctions". Similarly, Kultti (1999) points out that "even though auctions and posted prices turn out to be equivalent one rarely sees auction markets".

[3]The recent New Monetarist literature considers trading mechanisms and matching technologies as part of their environment; see for instance Lagos and Wright (2005).

[4]Thus, ours is a very slow-moving replacement process for buyers and sellers compared to that of Lu and McAfee (1996) and Kultti (1999).

where buyers and sellers play the price-posting and bargaining game infinitely many times. In our baseline model, sellers switch from one market to the other at a rate that is proportional to the payoff differential between the two markets, and we allow the adjustment process to take place at different speeds. In contrast, buyers cannot switch between markets. However, in the second part of the paper we allow both sellers and buyers to switch. We then explore the equilibrium properties of this environment and determine when a unique stable equilibrium exists.

As previously mentioned, via their informational requirements, we can relate bargaining and price posting to informal and formal market activities, respectively. This is because the informational requirements to implement price-posting and bargaining are quite different. This important distinction yields further insights on the co-existence of formal and informal markets. While the definition of informal economies is subject to some disagreement, there is never any debate that sellers in these markets strive to remain anonymous from taxing and regulatory authorities.[5] Informal sellers do not want to provide any public information about their locations and/or prices, in order to avoid the taxes and regulatory costs associated with formal markets. Thus, price posting is incompatible with informal activity, because it requires public observability of the sellers' terms of trade and locations. Instead, informal sellers must use a trading mechanism that provides limited public information about their whereabouts in order to remain hidden from the taxing and regulating authorities.[6] For this reason, informal markets resort to bargaining, which requires a minimal amount of public information when compared to price posting, as the terms of trade can always be renegotiated.[7] On the other hand, sellers trading in formal markets must publicly advertise their prices and locations to attract buyers.

To determine the robustness of the co-existence between formal and informal activities, we consider other differentiating features between these markets that have been previously highlighted in the literature. For instance, we consider the possibility of theft and confiscation in informal markets as well as different meeting technologies. We also explore the role of quality assurance when purchasing in formal sellers.[8] Within this environment, we find that when buyers cannot switch between formal and informal markets, some fraction of sellers will migrate from formal to informal markets whenever: (i) the formal sellers' advantage in providing quality assur-

[5]The vast majority of informal sector activities involves goods and services whose production and distribution are perfectly legal. The informal sector merely seeks to evade taxes and regulations. Clearly, illegal goods are also an important component of the informal economy. But we have chosen not to incorporate them, as the literature has not yet come to an agreement on how to model them. We refer to the reader to Feige (1990), De Soto (1989) and Portes et al. (1989), among others, for more on informal markets.

[6]It is important to note that informative advertising, which coincides with the inception of price-posting, is an essential feature for this trading institution to be effective. In price-posting sellers need to send informative signals describing their product, price and location in order to attract buyers as documented by Bagwell (2007).

[7]Auctions are clearly not suitable for informal markets as auctions are, by definition, public events.

[8]Note that since formal sellers are in fixed locations and are publicly observed by buyers and authorities, formal sellers can credibly provide such contracts. This practice is consistent with anti-lemon laws enforcing certain money-back guarantees prevalent in the formal markets in real life. In contrast, informal sellers cannot credibly provide such assurances to their customers as they do not legally exist.

ance erodes, (ii) the government imposes higher taxes and regulations in the formal market, (iii) the risk of crime and/or confiscation decreases in the informal market, and (iv) the number of buyers in the informal market increases.[9] These results may seem odd if one naively combined the results of Lu and McAfee (1996) and Kultti (1999), which would intuitively suggest that "posted price markets dominate bargaining markets". Our results differ because agents in our environment are replaced differently from the previous authors and also because we consider various additional features that distinguish formal and informal markets. These additional asymmetries between the two competing markets are not present in Lu and McAfee (1996) nor Kultti (1999). These extra differences (taxes, theft and quality assurance) are key in determining the resulting equilibria that delivers co-existence between the two trading protocols because they affect the seller's payoffs when trading in formal and informal markets. In particular, since we consider an evolutionary framework, the differential payoffs are crucial in determining the measure of sellers that trade in formal and informal markets over time.

When both sellers and buyers can switch between formal and informal markets, analytical results are not possible, and a numerical exercise is required to determine additional equilibrium properties. We find that, for a broad range of parameter values, if the net lump-sum cost for a seller in the formal sector (relative to that in the informal sector) has the opposite sign of the net lump-sum cost for a buyer in the formal sector (relative to that in the informal sector), then there exists a locally stable equilibrium in which formal and informal markets co-exist. If the two relative net lump-sum costs are both negative, then there exists a unique locally stable equilibrium where only formal activity takes place in the long run. In contrast, if the two relative net lump-sum costs are both positive, then there exists a unique locally stable equilibrium with only informal trade taking place.

The rest of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 develops the benchmark model, where sellers are mobile and buyers are fixed. Section 4 develops a more general model, where both buyers and sellers are mobile. Section 5 concludes. Appendix A contains the proofs of all results stated in the text. Appendix B describes another interpretation of our results, in a model of replicator/imitation dynamics. Appendix C provides an alphabetical index of all the notation used in the paper, while Appendix D contains some additional figures useful in interpreting the results from Section 4.

2 Related Literature

This paper relates to two different strands of literature. One literature explores the equilibrium obtained in frictional environments where various trading mechanisms are considered. The other one studies the equilibrium properties of economies with formal and informal markets.

[9] Conversely, sellers will switch from the informal to the formal market whenever the opposite changes occur.

Since Diamond (1971), much more attention has been paid to the explicit microstructure of markets and trading institutions. This increased attention has come from different perspectives. For instance, McAfee (1993), Peters (1994) and Peters (1995), among others, highlight that the institutions that govern trade themselves are endogenous. These authors explore how endogenous equilibrium institutions may emerge while using a mechanism design approach. When various trading protocols are considered, Arnold and Lippman (1995), Ehrman and Peters (1994), and Lu and McAfee (1996), Kultti (1999) and Julien et al. (2002), among others, compare different trading mechanisms to determine the advantages of these institutions and in which markets these are more suitable. These authors typically explore frictional environments, auctions and posted prices.[10] A robust result in this literature is that in large markets, price posting and auctions generate the same revenue and are efficient. In this paper, we complement this literature, by exploring when bargaining and price posting can co-exist in a large frictional and evolutionary environment.

Regarding theoretical frameworks that incorporate formal and informal activity, Rauch (1991) considers various forms of regulation avoidance and the emergence of the informal sector. Within the same spirit, Nicolini (1998) considers tax evasion as the main cause for informal activities to exist alongside formal markets.[11] Subsequent general equilibrium models employ existing estimates of the informal economy for calibration purposes. For instance, Koreshkova (2006) and Aruoba (2010) consider optimal taxation models and aim at rationalizing observed income and inflation tax rates for a cross section of countries. Meanwhile, Ordonez (2014) assess the quantitative effect of incomplete tax enforcement on aggregate output and productivity in Mexico. Typically, the general equilibrium analyses that study the co-existence of informal and formal markets share the assumption that these activities are different in nature. These differences are such that either the goods being produced in formal and informal markets are assumed to be different, as in Aruoba (2010), or the technologies used to produce the goods in both markets or the means of payment required to obtain the goods in formal and informal markets are assumed to be different, as in Koreshkova (2006), Antunes and Cavalcanti (2007), Amaral and Quintin (2006), Prado (2011) and D'Erasmo and Boedo (2012). This literature, however, has not fully explored how the co-existence of formal and informal activity is affected when there are different trading mechanisms, matching technologies and informational requirements. Notable exceptions are Aruoba (2010) and Gomis-Porqueras et al. (2014), who consider an environment where firms

[10] The use of auctions has traditionally been motivated by the existence of a monopoly seller possessing imperfect information about the buyers' valuations of the object for sale. On the other hand, price-posting has usually been motivated by the sale of goods whose value is commonly understood, no bidding takes place, and the good may be rationed according to some rule.

[11] In particular, Rauch (1991) considers an environment with a minimum wage policy that can only be enforced for sufficiently large firms. This results in smaller firms operating in the informal sector and paying lower wages. Meanwhile, Nicolini (1998) shows how inflation is optimal when tax evasion in the informal market is widespread, as income generated by cash is difficult to monitor by tax authorities.

produce goods in different markets while using different trading protocols. We complement these papers.

3 Benchmark Model

Consider an economy with a large number of buyers and sellers. Time is continuous. At any point in time, capacity-constrained sellers produce at most one indivisible unit of a homogeneous perishable good. Thus, it is not possible for sellers to accumulate an inventory of unsold goods. Buyers can purchase this good in either formal or informal markets. Moreover, buyers can only visit one seller at a given point in time. However buyers, before deciding which market to participate in, cannot coordinate among themselves to minimize the possibility of being rationed.[12]

Formal buyers trade in a market with posted terms of trade where all buyers can see their advertised prices and locations. In contrast, informal sellers trade in markets where the trading protocol is bargaining. As a benchmark, we assume that buyers are exogenously divided between formal and informal markets. Let b^{fo} (b^{in}) be the ratio of buyers in the formal (informal) market vis-à-vis the total number of sellers in *both* markets; thus, $b^{\text{fo}} + b^{\text{in}} = b$ where b denotes the ratio of buyers to sellers in the economy as a whole. In contrast to buyers, *sellers* can switch between formal and informal markets. Let s^{fo} (s^{in}) denote the fraction of sellers in the formal (informal) market. Thus we have that $s^{\text{fo}} + s^{\text{in}} = 1$. These fractions can change over time, as the relative payoffs for sellers in formal and informal markets differ.[13]

The simplifying assumption that only sellers can switch between formal and informal markets allows us to obtain analytical solutions. It also reflects the fact that the predominant factor for sellers in deciding where to locate their business is to be close to buyers. This is not the case for most buyers, where accessibility to sellers is not of first order importance. Households also take into account other factors (such as access to the workplace, schools and commuting costs) when deciding where to locate. Given these considerations, we can think of buyers as being streamed into one market or the other on the basis of exogenous factors. In contrast, sellers' decision of where to location and produce is endogenous and strategic.

In the next subsection, we describe the underlying preferences of buyers and sellers, the characteristics that distinguish formal and informal markets, as well as the different trading mechanisms and matching technologies.

[12]Note that when more than one buyer visits a seller, the seller needs to select which buyer to sell the good. For more details we refer to Burdett et al. (2001).

[13]In an alternative interpretation of our model, there are no separate populations of formal and informal agents. *All* agents participate in both formal and informal markets, where informal transactions are cash-only. In this interpretation, s^{fo} (s^{in}) denotes the fraction of each seller's transactions which are formal (informal). The equilibria in our model then correspond to mixed strategy Nash equilibria. See Remarks 1 and 2 for details.

3.1 Preferences

Buyers and sellers have quasilinear preferences. A buyer in the formal (informal) market obtains a value of $v_{\rm bu}^{\rm fo}$ ($v_{\rm bu}^{\rm in}$) when she consumes a unit of the good.[14] Thus, if the formal (informal) market price of the good is $p^{\rm fo}$ ($p^{\rm in}$), then the formal (informal) buyer obtains a total payoff of $v_{\rm bu}^{\rm fo} - p^{\rm fo}$ ($v_{\rm bu}^{\rm in} - p^{\rm in}$) after trade has taken place. On the other hand, formal (informal) sellers incur a cost of $c_{\rm se}^{\rm fo}$ ($c_{\rm se}^{\rm in}$) per unit of good produced, which includes both labor and other input costs. Thus, the per period total payoff of formal (informal) sellers is $p^{\rm fo} - c_{\rm se}^{\rm fo}$ ($p^{\rm in} - c_{\rm se}^{\rm in}$).[15]

In order for trades to occur in both markets, individual rationality for both buyers and sellers has to be satisfied. This requires that $c_{\rm se}^{\rm fo} \leq p^{\rm fo} \leq v_{\rm bu}^{\rm fo}$ and $c_{\rm se}^{\rm in} \leq p^{\rm in} \leq v_{\rm bu}^{\rm in}$ always have to hold. Let $g^{\rm fo} := v_{\rm bu}^{\rm fo} - c_{\rm se}^{\rm fo}$ denote the "gains from trade" in the formal market, while $g^{\rm in} := v_{\rm bu}^{\rm in} - c_{\rm se}^{\rm in}$ represent the total "gains from trade" in the informal market. Without loss of generality, we normalize the buyer and seller's utility functions so that $g^{\rm fo} = 1$.[16] If agents do not trade, then buyers and sellers obtain a zero payoff.

3.2 Formal versus Informal Markets: Further Differences

Apart from their trading mechanisms, formal and informal markets differ along other dimensions. The literature on formal and informal markets has emphasized other features that are quite different across these markets.[17] For instance, formal sellers are taxed, have to comply with regulations and typically offer quality assurance when selling their products. Meanwhile, informal sellers are more susceptible to face theft, bribery, fines and confiscations.[18] In the next subsections we elaborate how these other distinguishing features of formal and informal markets affect the co-existence of formal and informal markets.

3.2.1 Taxation, Regulation, Confiscation and Theft

As in Nicolini (1998) Aruoba (2010) and Gomis-Porqueras et al. (2014), among others, we assume that formal sellers face a profit tax.[19] Let us denote the effective tax rate for a formal seller as $T_{\rm se}^{\rm fo}$. In the informal sector, sellers do not pay taxes nor incur any regulation cost. However, informal sellers face the possibility that their part of their profits be taken away from the government (through a fine) or by criminals (through theft). As in Prado (2011), these costs are assumed to

[14] In Section 3.2, we will explain why, in general, $v_{\rm bu}^{\rm in} < v_{\rm bu}^{\rm fo}$.

[15] In Section 3.2, we will explain why, in general, $c_{\rm se}^{\rm in} < c_{\rm se}^{\rm fo}$.

[16] In Section 3.2, we will see that, in general, $g^{\rm in} < g^{\rm fo}$.

[17] Feige (1990), De Soto (1989) and Portes et al. (1989), among others, provide a detailed description of the differences between informal and formal markets.

[18] We refer to the reader to Putnins and Sauka (2015) and Chmielowski (2015) for more on these issues.

[19] Note the profit tax is equivalent to a value added tax on the sale of the goods.

be proportional to informal earnings.[20] These payments effectively function as a form of "income tax" on the informal sector. Let $T_{\text{se}}^{\text{in}}$ represent the "effective tax rate" of operating in the informal economy.[21]

3.2.2 Quality Assurance

An important distinguishing feature of formal markets relative to informal ones —and one which has not previously been emphasized by the literature— is the provision of quality assurance.[22] Since formal sellers are registered and monitored by government authorities, they can credibly write contracts that provide warranties to their customers. In contrast, informal sellers *cannot* credibly offer such quality assurance, as informal market transactions *do not exist* within the confines of the legal system.

To incorporate this aspect, let us consider uncertainty regarding the quality of the product purchased by the buyer. In particular, we assume that all sellers have access to the same technology, so that the probability of faulty goods is the same in formal and informal markets. Let $c_{\text{se}}^{\text{in}}$ be the unit cost of sellers producing in the informal market, i.e., the unit cost of sellers producing the good without any quality assurance. Let q be the per unit cost that each formal seller incurs when providing quality assurance. Thus, with quality assurance, the unit cost of production for *formal* sellers becomes $c_{\text{se}}^{\text{fo}} = c_{\text{se}}^{\text{in}} + q$. To simplify exposition, we assume that formal sellers pass all of the quality assurance costs q onto the buyer.

When a good is defective, it provides less utility to the buyer. Thus, if $v_{\text{bu}}^{\text{in}}$ is the expected utility to the buyer of the good purchased in the informal market, then $v_{\text{bu}}^{\text{in}} < v_{\text{bu}}^{\text{fo}}$. The total value for the buyer of purchasing the good in the formal market is denoted by $v_{\text{bu}}^{\text{fo}} = v_{\text{bu}}^{\text{in}} + \alpha(q)$, where $\alpha(q)$ is the benefit that each formal buyer receives from quality assurance. We assume that α is increasing, differentiable, and concave function of q, with $\alpha(0) = 0$.[23]

Lemma A *If $\alpha(\cdot)$ is increasing, differentiable, and concave, and formal sellers provide the efficient level of quality assurance, then $g^{\text{in}} \leq g^{\text{fo}}$.*

The trading protocols employed in the formal and informal markets specify how the gains from trade are split between the buyer and the seller. Thus, the fact $g^{\text{in}} \leq g^{\text{fo}}$ implies that there is generally a larger surplus to be divided in the formal market, so that both buyer and seller

[20] This can capture the probability that an informal seller's profit can be stolen or their merchandise be confiscated by government authorities.

[21] We refer to Donovan (2008) for more on the costs of operating in the informal market.

[22] This can take many forms such as free repair/replacement, a full money-back guarantee, on-site customer service, twenty-four hour telephone customer assistance, and/or cash compensation for unsatisfactory product performance.

[23] Concavity is a very reasonable assumption in this context, because α is generally bounded above: $\lim_{q\to\infty} \alpha(q) = v^* - v_{\text{bu}}^{\text{in}}$, where v^* is the value of consuming a "perfect" commodity, with no defects upon repair or replacement. Note that given the concavity of α and the linearity of $c_{\text{se}}^{\text{fo}}$ in q , there is an optimal amount of q, which may lead to a less-than-full replacement or less-than-perfect repair.

can potentially be better off. This, in turn, implies that the government can tax a fraction of up to $T_0 = g^{\text{fo}} - g^{\text{in}}$ without driving participants into the informal market.[24]

3.3 Trading Mechanisms and Matching Technologies

Below we provide details regarding the formal and informal markets' respective matching technologies and trading protocols.

3.3.1 Formal Markets

In order to capture the informational requirements of formal sellers, we use the price-posting and directed-search framework of Burdett et al. (2001). In the formal market, the *ex ante* identical sellers each have a precise location at any point in time. In order to attract buyers, these formal sellers advertise their prices and location. The information contained in these advertisements are costlessly seen by all relevant *ex ante* identical uncoordinated buyers.

Since sellers compete for buyers, and these buyers cannot coordinate which seller to visit, sellers and buyers in the formal market play a strategic game of complete information composed of three stages. In the first stage, sellers simultaneously, independently and costlessly advertise a single posted price as well as their location.[25] In the second stage, buyers observe prices, and simultaneously and independently choose which seller(s) to visit. In the third stage, matches are realized and trade takes place. As in Burdett et al. (2001), we assume that, after visiting one seller, buyers find it prohibitively costly to search again within the same period. So, for any given period of time, each buyer can visit only *one* seller, but a seller can be visited by multiple buyers. In that case, the seller sells his product to a randomly chosen buyer.

Note that, in this environment, formal buyers are more likely to visit a formal seller with the lowest posted price. But since buyers are not coordinated, they may face more competition at these cheaper locations. If multiple buyers choose to visit the same seller, then only *one* of the buyers can purchase the good, while the rest of buyers receive a payoff of 0. On the other hand, if *no* buyers visit a seller, then he cannot sell his good, so he receives a payoff of 0.[26]

Since all buyers are *ex ante* identical, we focus on the symmetric equilibrium where buyers use the same mixed strategy when deciding which seller(s) to visit. Likewise, since all sellers are

[24] All buyers have identical preferences. If we relaxed this assumption and considered buyer heterogeneity in terms of income or preferences, then those who are poorer, less risk-averse, and/or have a higher discount rate would prefer the cheaper but less reliable goods of the informal market. This, buyer heterogeneity alone could explain the co-existence of the two markets. Introducing heterogeneity amongst buyers (or sellers) would only strengthen our conclusions as there is more scope for co-existence.

[25] We refer to Gomis-Porqueras et al. (2017) for the properties of equilibrium under price posting and auctions in environments where advertising is costly and its reach is probabilistic.

[26] In contrast to Camera and Selcuk (2009), here prices posted by sellers cannot be renegotiated depending on market conditions, so that there is no distinction between the posted list price and the sale price

also *ex ante* identical, they all use the same pricing strategy. The next theorem summarizes the main results of Burdett et al. (2001).

Theorem BSW *Let m be the total number of sellers in the formal market, and let B_f be the ratio of buyers to sellers in the formal market (so there are $B_f m$ buyers). There is a unique symmetric Nash equilibrium of the formal market game where all sellers post an identical price, p, and all buyers randomly visit all sellers with equal probability. Let Φ be the probability that any given seller sells his product (i.e. is visited by at least one buyer), and let Ω be the probability that any given buyer purchases the good. Then p, Φ, and Ω are entirely determined by B_f and m. Furthermore, if we let $m \to \infty$ while holding B_f fixed, then we get:*

$$p(B_f) := \lim_{m\to\infty} p(B_f, m) \;=\; c^{\text{fo}}_{\text{se}} + u^{\text{fo}}_{\text{se}}(B_f), \tag{1}$$

$$\textit{where} \quad u^{\text{fo}}_{\text{se}}(B_f,) := 1 - \frac{B_f}{e^{B_f} - 1}. \tag{2}$$

$$\textit{Also,} \quad P^{\text{fo}}_{\text{se}}(B_f) := \lim_{m\to\infty} \Phi(B_f, m) \;=\; 1 - e^{-B_f}, \tag{3}$$

$$\textit{and} \quad P^{\text{fo}}_{\text{bu}}(B_f) := \lim_{m\to\infty} \Omega(B_f, m) \;=\; \frac{P^{\text{fo}}_{\text{se}}(B_f)}{B_f}. \tag{4}$$

Note that $P^{\text{fo}}_{\text{se}}(B_f)$ ($P^{\text{fo}}_{\text{bu}}(B_f)$) represents the probability that at any point in time any seller (buyer) is able to sell (buy) the good. At any given point in time, if a seller makes a sale in the large formal market ($m \to \infty$), then his pre-tax payoff is given by $u^{\text{fo}}_{\text{se}}(B_f)$; otherwise his payoff is 0. Thus, the seller's pre-tax *expected* payoff in the large formal market game is $U^{\text{fo}}_{\text{se}}(B_f) \;:=\; P^{\text{fo}}_{\text{se}}(B_f) \cdot u^{\text{fo}}_{\text{se}}(B_f)$ where $B_f = b^{\text{fo}}/s^{\text{fo}}$.[27]

Given that the proportional tax rate paid by formal sellers is $T^{\text{fo}}_{\text{se}}$, the seller's expected *after-tax* payoff in the formal market is given by:

$$\begin{aligned}
\widetilde{U}^{\text{fo}}_{\text{se}}(s^{\text{fo}}) &:= (1 - T^{\text{fo}}_{\text{se}})\, U^{\text{fo}}_{\text{se}}\left(\frac{b^{\text{fo}}}{s^{\text{fo}}}\right) \\
&= (1 - T^{\text{fo}}_{\text{se}}) \left(1 - \exp\left(\frac{-b^{\text{fo}}}{s^{\text{fo}}}\right)\right) \left(1 - \frac{b^{\text{fo}}/s^{\text{fo}}}{\exp(b^{\text{fo}}/s^{\text{fo}}) - 1}\right).
\end{aligned} \tag{5}$$

Note that we write $\widetilde{U}^{\text{fo}}_{\text{se}}$ as a function of s^{fo} only, because in the benchmark model, b^{fo} is fixed.

[27]Recall that b^{fo} is the ratio of buyers in the formal market relative to the total number of sellers in *both* markets, while s^{fo} is the fraction of sellers in the formal market to the total number of sellers in both markets. Thus, we have that $B_f = b^{\text{fo}}/s^{\text{fo}}$.

3.3.2 Informal Markets

Informal sellers cannot publicly advertise their exact locations and prices, or remain fixed in one location, because they are trying to avoid government detection. So instead, would-be buyers in the informal market must search in locations where informal sellers are known to congregate (e.g. certain street corners, parking lots, and parks), and hope to randomly encounter one.[28] We therefore assume that in the informal market, buyers are randomly matched with sellers.

As in the formal sector, each informal buyer can visit only *one* seller per period, and buyers cannot coordinate which seller to visit. Informal buyers search informal sellers with the matching probabilities as in the directed search model of Burdett et al. (2001). Thus, if $B_i := b^{\text{in}}/s^{\text{in}}$ is the ratio of buyers to sellers in the informal market, then equation (3) in *Theorem BSW* implies that the probability that any given informal seller makes a sale during any given period is given by

$$P^{\text{in}}_{\text{se}}(B_i) \quad = \quad 1 - e^{-B_i}. \tag{6}$$

Instead of trading at publicly posted prices, the informal seller and buyer negotiate a price through bargaining, thereby splitting the total surplus, g^{in}. Suppose that the informal seller receives a fraction $\eta(B_i) \in [0, 1]$ of this surplus, while the informal buyer receives the remaining fraction $1 - \eta(B_i)$.[29] Thus, a matched informal buyer then receives a payoff equal to $u^{\text{in}}_{\text{se}} := \eta(B_i)\, g^{\text{in}}$. Then the resulting pre-theft/confiscation cost expected payoff for sellers in the informal market is given by:

$$U^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right) \;:=\; u^{\text{in}}_{\text{se}} \cdot P^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right) \;=\; g^{\text{in}}\, \eta\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right) P^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right). \tag{7}$$

Clearly, the higher the ratio of buyers to sellers in the informal market, the stronger each seller's negotiating position or bargaining power becomes, and the better each seller will do in bilateral bargaining. In the limit when there are infinitely many buyers for every seller, the sellers will capture *all* of the surplus in the informal market. Thus, the seller's bargaining power η is an increasing function, such that:[30]

$$\lim_{B_i \to \infty} \eta(B_i) \;=\; 1. \tag{8}$$

In our benchmark buyers cannot switch between informal and formal markets, thus b^{in} is a

[28] Informal buyers have some vague notion where informal sellers congregate. Once they arrive at such location there is random search among informal sellers. We refer to Donovan (2008) for more on the characteristics of street vendors working in the underground economy.

[29] $\eta(B_i)$ is a function of B_i (the ratio of buyers to sellers), which will determine the relative bargaining power of a buyer and a seller. Later, in Section 4.2, we will present one possible model of this surplus division process, but there is no need to commit to a specific model here.

[30] It would also be reasonable to assume $\lim_{B_i \searrow 0} \eta(B_i) = 0$. But this is unnecessary for our analysis.

constant. In contrast, sellers are able to switch between markets at any point in time. Since $s^{\text{in}} = 1 - s^{\text{fo}}$, we can regard the informal seller's utility as a function of s^{fo} only. Thus, the relevant expect payoff for the informal sellers after taking into account their confiscation/theft cost is given by:

$$\widetilde{U}^{\text{in}}_{\text{se}}(s^{\text{fo}}) \;:=\; (1 - T^{\text{in}}_{\text{se}})\, U^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{1 - s^{\text{fo}}}\right) \;=\; (1 - T^{\text{in}}_{\text{se}})\, g^{\text{in}}\, \eta\left(\frac{b^{\text{in}}}{1 - s^{\text{fo}}}\right)\left(1 - \exp\left(\frac{-b^{\text{in}}}{1 - s^{\text{fo}}}\right)\right). \quad (9)$$

3.4 Dynamic Equilibrium

In the previous sections we have specified the payoffs of buyers and sellers in the formal and informal markets for a given point in time. To explore how the informal and formal markets evolve over time, we need to specify how agents will adjust and consequently how their payoffs will change over time. Here we consider this dynamic link between periods via our main equilibrium concept, 'best response dynamics', where agents who can switch between markets will migrate from one market to the other at a rate that is proportional to their market payoff differential. More precisely, let $s^{\text{fo}}(t)$, and $s^{\text{in}}(t)$ represent the populations of formal/informal sellers at time t, and let $\dot{s}^{\text{fo}}(t)$ and $\dot{s}^{\text{in}}(t)$ represent the derivatives of these functions at time t.[31] Then we have that the evolution of these populations is as follows:[32]

$$\dot{s}^{\text{fo}}(t) \quad = \quad -\dot{s}^{\text{in}}(t) \quad = \quad \lambda_{\text{se}}\left(\widetilde{U}^{\text{fo}}_{\text{se}}\left(s^{\text{fo}}(t)\right) - \widetilde{U}^{\text{in}}_{\text{se}}\left(s^{\text{fo}}(t)\right)\right), \quad (10)$$

where $\widetilde{U}^{\text{in}}_{\text{fo}}(s^{\text{fo}})$ and $\widetilde{U}^{\text{in}}_{\text{se}}(s^{\text{fo}})$ are given by equations (5) and (9), while $\lambda_{\text{se}} : \mathbb{R} \longrightarrow \mathbb{R}$ is a strictly increasing function that modulates the speed of adjustment and satisfies $\lambda_{\text{se}}(0) = 0$.[33] If migration from the formal to informal sector is exactly as difficult and costly as migration from the informal to the formal sector, then λ_{se} will be an odd function.[34] However, if migrating in one direction is more difficult than migrating in the other direction, then λ_{se} will not be an odd function.[35]

[31] Formally, in a continuous-time model, where t ranges over the set of real numbers, we would define $\dot{s}^{\text{fo}} := \mathrm{d}s^{\text{fo}}/\mathrm{d}t$ and $\dot{s}^{\text{in}} := \mathrm{d}s^{\text{in}}/\mathrm{d}t$. In a discrete-time model, where t ranges over the set of integers, we would define $\dot{s}^{\text{fo}}(t) := s^{\text{fo}}(t+1) - s^{\text{fo}}(t)$ and $\dot{s}^{\text{in}}(t) := s^{\text{in}}(t+1) - s^{\text{in}}(t)$. Thus, the dynamical equation (10) admits both a continuous-time and a discrete-time interpretation. The equilibrium characterization of Theorem 1 holds in both cases.

[32] The expected utility of *buyers* in the informal market is irrelevant to the dynamics of this model, because we have assumed that they cannot switch from informal to informal markets.

[33] Typically, λ_{se} is just multiplication by a positive constant.

[34] That is: $\lambda_{\text{se}}(-r) = -\lambda_{\text{se}}(r)$, for all real numbers r.

[35] If λ_{se} is an odd function, then the dynamical system converges to equilibrium just as quickly from either direction. Thus, the informal market would show a symmetric response to tax increases and tax decreases, as found by Christopoulos (2003) in Greece. To see how λ_{se} might not be odd, note that it might cost more for a seller to switch from the informal market to the formal market than vice versa (e.g. because of the need to acquire licenses, rent a retail location, etc.); this would be reflected by having $|\lambda_{\text{se}}(r)| < |\lambda_{\text{se}}(-r)|$ for any given $r > 0$. This is consistent with empirical findings by Giles et al. (2001) and Wang et al. (2012) in Taiwan and New Zealand, respectively.

An equilibrium of an economy where sellers can decide to produce in the formal and informal markets is a fixed point of the dynamical system represented by equation (10). Given a fixed fraction of buyers participating in the formal and informal market (b^{fo} and b^{in} respectively), sellers will migrate between the two markets at a rate which is proportional to the payoff differential between the two markets (of which speed of adjustment is modulated by λ_{se}), until their payoffs from both markets are the same. Thus, the economy is in *equilibrium* if and only if $s^{\text{fo}} = s^*$ is a value such that:[36]

$$\widetilde{U}^{\text{fo}}_{\text{se}}(s^*) \quad = \quad \widetilde{U}^{\text{in}}_{\text{se}}(s^*). \tag{11}$$

Remark 1. Although we have derived equation (11) using an evolutionary model, it could also be interpreted as an uncorrelated, symmetric mixed-strategy Nash equilibrium. This interpretation assumes a static environment, where each seller plays the mixed strategy $(s^{\text{fo}}, s^{\text{in}})$.[37] This could mean that she randomly chooses whether to participate in the formal or informal market according to the probabilities $(s^{\text{fo}}, s^{\text{in}})$. Or it could mean that she spends a fraction s^{fo} of her time in the formal market and the remaining fraction s^{in} in the informal market. In any case, the sellers cannot coordinate, so there is no correlation between their strategies. The result is that, at any point in time, a fraction s^{fo} of sellers can be found in the formal market, while the remaining fraction s^{in} are in the informal market. Equation (11) is then equivalent to saying that this profile of mixed strategies is a Nash equilibrium, as in Camera and Delacroix (2004) or Michelacci and Suarez (2006).

There is also a third alternative framework that leads to the equilibrium represented by equation (11), namely *replicator/imitation dynamics*. We refer the reader to Appendix B for further details.

3.5 Existence and Properties of the Equilibrium

Let us define $\widetilde{U}(s^{\text{fo}}) := \widetilde{U}^{\text{fo}}_{\text{se}}(s^{\text{fo}}) - \widetilde{U}^{\text{in}}_{\text{se}}(s^{\text{fo}})$; this represents the sellers' net gain from being in the formal market relative to the informal market. Note that the equilibrium condition, given by equation (11), is equivalent to $\widetilde{U}(s^*) = 0$.

We say that an equilibrium s^* is *locally stable* if $\widetilde{U}'(s^*) < 0$. We will say that s^* is a *mixed market equilibrium* if it has both formal and informal market of nontrivial size —that is, $0 < s^* < 1$. The main result of this section is the following.

Theorem 1 *If we satisfy the following conditions*

$$1 - \frac{b^{\text{fo}} + 1}{\exp(b^{\text{fo}})} \quad < \quad \frac{(1 - T^{\text{in}}_{\text{se}})\, g^{\text{in}}}{(1 - T^{\text{fo}}_{\text{se}})} \quad < \quad \frac{1}{\eta(b^{\text{in}})\ (1 - \exp(-b^{\text{in}}))},$$

[36] Throughout this paper, we use the term "equilibrium" to mean a *stationary* equilibrium —that is, one which is unchanging over time.

[37] All sellers are *ex ante* identical, so they all play the same mixed strategy.

then there exists a locally stable mixed market equilibrium $s^* \in (0,1)$. Furthermore, the equilibrium fraction of formal sellers, s^*, has the following properties: ***(i)*** *s^* is decreasing as a function of g^{in}, $T_{\text{se}}^{\text{fo}}$, and b^{in}; and* ***(ii)*** *s^* is increasing as a function of $T_{\text{se}}^{\text{in}}$ and b^{fo}.*

Recall that in the benchmark model b^{fo} and b^{in} are exogenous constants in this model. If $b^{\text{fo}} = 0$, then the equilibrium described in Theorem 1 yields $s^{\text{fo}} = 0$. Likewise, if $b^{\text{in}} = 0$, then we will have $s^{\text{in}} = 0$. However, if b^{fo} and b^{in} are both nonzero, then s^{fo} and s^{in} will also be nonzero in equilibrium. Thus, for a broad range of parameters values, formal and informal markets of nontrivial size will co-exist in a stable equilibrium. Note that as long as there are some buyers in both formal and informal markets, all sellers cannot find it in their interest to settle in only of one of the markets regardless of g^{in}, $T_{\text{se}}^{\text{fo}}$ and $T_{\text{se}}^{\text{in}}$. This is the case as there would be some sellers that find profitable to switch to the other market with no sellers and being able to serve the buyers in that market.

Theorem 1 also highlights what makes sellers move between markets and in which direction. Part (ii) of Theorem 1 states that sellers will tend to migrate from the formal market to the informal market if the formal market's advantage in quality assurance erodes (g^{in} increases relative to g^{fo}), or the government imposes higher taxes and regulations ($T_{\text{se}}^{\text{fo}}$ increases), or more buyers migrate to the informal market (b^{in} increases). Conversely, sellers will migrate from the informal market back to the formal market whenever the opposite changes occur in these parameters. Likewise, sellers will migrate to the formal market if the risk of crime and/or confiscation increases in the informal market (i.e. $T_{\text{se}}^{\text{in}}$ increases) or if buyers migrate to the formal market (b^{fo} increases).[38] The discrepancy between our results and the earlier results of Lu and McAfee (1996) and Kultti (1999) is driven partly by the different agent-replacement dynamics in our model, and partly because we explicitly model the way in which taxes, theft and quality assurance affect the sellers' payoffs when trading in formal and informal markets.

4 Both Buyers and Sellers Moving across Markets

Now we consider an environment where both sellers and buyers can switch among formal and informal markets. In this new environment, the buyer populations, b^{in} and b^{fo}, and seller populations, s^{in} and s^{fo}, are endogenous and can change over time. As in Section 3, we will also consider factors other than the trading mechanisms that distinguish formal and informal markets. After describing the equilibria of this model (Result 2), we discuss corresponding the equilibrium properties.

[38] One can also imagine circumstances where it is the *buyers* who are mobile, while the sellers are fixed —for instance, tourists arriving at New York City who know where to find all formal and informal sellers. In that case, it would not be difficult to see that the above result (and comparative statics) would still hold.

4.1 Formal Markets

As in Section 3.3.1, the behaviour and payoffs of buyers and sellers in the formal market are summarized in Theorem BSW, but the behaviour and payoffs of buyers and sellers in the informal market need to be re-derived.

Recall that $B_f = b^{\text{fo}}/s^{\text{fo}}$ is the ratio of buyers to sellers in the formal market. Then a formal seller's pre-tax expected utility is again given by $U^{\text{fo}}_{\text{se}}(B_f) \; := \; P^{\text{fo}}_{\text{se}}(B_f) \cdot u^{\text{fo}}_{\text{se}}(B_f)$, where $P^{\text{fo}}_{\text{se}}(B_f)$ and $u^{\text{fo}}_{\text{se}}(B_f)$ are defined in equations (2) and (3).

Let $p(B_f)$ be the formal market equilibrium price from equation (1). If a formal market buyer makes a purchase, then her payoff is given by:

$$\begin{aligned} u^{\text{fo}}_{\text{bu}}(B_f) := v^{\text{fo}}_{\text{bu}} - p(B_f) \quad &= \quad v^{\text{fo}}_{\text{bu}} - c^{\text{fo}}_{\text{se}} - u^{\text{fo}}_{\text{se}}(B_f) \\ = g^{\text{fo}} - u^{\text{fo}}_{\text{se}}(B_f) \quad = \quad 1 - u^{\text{fo}}_{\text{se}}(B_f) \quad &= \quad \frac{B_f}{e^{B_f} - 1}. \end{aligned} \tag{12}$$

If a buyer doesn't make a purchase then her payoff is zero. Thus, her *expected* payoff for participating in the large formal market game is $U^{\text{fo}}_{\text{bu}}(B_f) \; := \; P^{\text{fo}}_{\text{bu}}(B_f) \cdot u^{\text{fo}}_{\text{bu}}(B_f)$, where $P^{\text{fo}}_{\text{bu}}(B_f)$ is defined in equation (4).

4.2 Informal Markets

As in the formal market, we assume that informal sellers have fixed locations, and informal buyers visit them.[39] As in the previous section, the matching probabilities and tie breaking rule are given by Burdett et al. (2001). Recall that, if $B_i := b^{\text{in}}/s^{\text{in}}$ is the ratio of buyers to sellers in the informal market, then the probability for a given seller to be visited by at least one buyer during at any point in time is obtained by replacing B_f with B_i in equation (3), to obtain $P^{\text{in}}_{\text{se}}(B_i) \quad = \quad 1 - e^{-B_i}$. Likewise, the probability of a given buyer obtaining the goods from the one seller he visits is given by:

$$P^{\text{in}}_{\text{bu}}(B_i) \quad = \quad \frac{P^{\text{in}}_{\text{se}}(B_i)}{B_i}. \tag{13}$$

In Section 3.2.1, we assume that the informal seller and buyer negotiate to split the surplus according to proportions $(\eta, 1-\eta)$, where η depended on the ratio of buyers to sellers in this market.[40] Now, however, we need to model the negotiation process more explicitly, as the outside options of buyers and sellers are relevant. This is the case as both buyers and sellers can switch

[39] In fact, this assumption is not critical; we could instead reverse the roles of buyers and sellers in the informal market, so that it is the informal *buyers* who have fixed locations (home or workplace), and informal *sellers* who visit them. This would correspond to the door-to-door selling strategy used by informal sellers in some developing countries. If we define $S_i := s^{\text{in}}/b^{\text{in}}$ (i.e. the ratio of sellers to buyers in the informal market), then we obtain $P^{\text{in}}_{\text{bu}}(S_i) = 1 - e^{-S_i}$ and $P^{\text{in}}_{\text{se}}(S_i) = \frac{P^{\text{in}}_{\text{bu}}(S_i)}{S_i}$. This alternative model yields results which are qualitatively identical to the results we present here.

[40] It was not necessary to be more specific about the negotiation process in order to obtain Theorem 1.

markets, thus changing the surpluses that they can obtain. The trading mechanisms we use in the informal market are the solution concepts of the Nash bargaining framework. Given our assumptions about the utility functions of the buyer and seller, the (bargaining) set of feasible utility allocations is the convex hull Δ of the points $(0,0)$, $(g^{\text{in}},0)$, and $(0,g^{\text{in}})$, where g^{in} is the total gains from trade to be divided in the informal market. The Pareto frontier of Δ is the diagonal line from $(g^{\text{in}},0)$ to $(0,g^{\text{in}})$. Thus, the Nash bargaining, the egalitarian bargaining and Kalai-Smorodinsky bargaining solutions all yield the same outcome, which is the midpoint of the diagonal line from $(g^{\text{in}},0)$ to $(0,g^{\text{in}})$.[41] Thus, it does not matter which particular bargaining solution we use; the bargaining outcome is robust to any of the aforementioned trading protocols.

Within the Nash bargaining framework, the bargaining outcome is determined by the disagreement payoffs of the two parties, namely the outside options available to buyers and sellers. To endogenize these disagreement payoffs, we assume that agents in the informal market are repeatedly and randomly matched to counter-parties (as in Burdett et al. (2001)), with whom they bargain. Importantly, we assume that this matching-and-bargaining process occurs on a much faster timescale than the ones corresponding to agents' consumptions and productions. We will refer to each matching-and-bargaining period as a "turn", and each production-and-consumption period as a "chapter". We assume that there are many turns per chapter. For instance, it may be that each seller (buyer) can produce (consume) one unit per day, but they can meet and bargain with a new counter-party every 15 minutes. Thus, assuming agents trade for 12 hours per day, there would be almost 50 turns per chapter.

In reality, each chapter will have a finite (but large) number of turns. But for simplicity, we will assume there are infinitely many turns per chapter. If the bargaining during a particular turn is successful, then both buyer and seller immediately enjoy the gains from trade. They exit the informal market for the rest of that chapter, and are replaced by a new buyer and seller (perhaps agents who have just completed the production/consumption they began during the previous chapter).[42] However, if bargaining is *not* successful, then both agents must reenter the informal market during the next turn and find a new counter-party to bargain with. This will allow us to compute the disagreement payoffs of the two parties recursively.

Formally, let $U^{\text{in}}_{\text{se}}$ be the expected payoff for a seller participating in the informal market during a particular turn, and let $U^{\text{in}}_{\text{bu}}$ be the expected payoff for a buyer participating in the informal market during that turn. Note that $U^{\text{in}}_{\text{se}}$ and $U^{\text{in}}_{\text{bu}}$ are functions of $B_i := b^{\text{in}}/s^{\text{in}}$, the ratio between informal buyers and sellers. Let $\delta \in (0,1)$ be the per-turn discount factor for all agents. If bargaining breaks down during one turn, then both parties must re-enter the informal market

[41] The Nash solution selects the point in Δ which maximizes the *product* of the buyer's and seller's utilities. The egalitarian solution maximizes the *minimum* of the two utilities, and the Kalai-Smorodinsky solution maximizes the minimum utility after both utilities have been rescaled, ranging from 0 to 1. In the domain Δ, it is clear that the maximizers of all three trading protocols coincide with the midpoint of the diagonal line from $(g^{\text{in}},0)$ to $(0,g^{\text{in}})$. Indeed, this is true for *any* Pareto-efficient bargaining solution that respects the axiom of Symmetry.

[42] Thus, the population of buyers and sellers in the informal market does not change over the course of a chapter.

during the next turn. Thus, the outside option for the seller is $\delta\, U^{\mathrm{in}}_{\mathrm{se}}(B_i)$, while the outside option for the buyer is $\delta\, U^{\mathrm{in}}_{\mathrm{bu}}(B_i)$. Therefore, the Nash bargaining solution awards the seller a payoff of $u^{\mathrm{in}}_{\mathrm{se}}(B_i)$ and the buyer a payoff of $u^{\mathrm{in}}_{\mathrm{bu}}(B_i)$, where these are given by:[43]

$$\begin{aligned} u^{\mathrm{in}}_{\mathrm{se}}(B_i) &= \frac{\delta\, U^{\mathrm{in}}_{\mathrm{se}}(B_i) + g^{\mathrm{in}} - \delta\, U^{\mathrm{in}}_{\mathrm{bu}}(B_i)}{2}; \\ \text{and} \quad u^{\mathrm{in}}_{\mathrm{bu}}(B_i) &= \frac{\delta\, U^{\mathrm{in}}_{\mathrm{bu}}(B_i) + g^{\mathrm{in}} - \delta\, U^{\mathrm{in}}_{\mathrm{se}}(B_i)}{2}. \end{aligned} \tag{14}$$

Recall, however, that $U^{\mathrm{in}}_{\mathrm{se}}(B_i) = P^{\mathrm{in}}_{\mathrm{se}}(B_i)\, u^{\mathrm{in}}_{\mathrm{se}}(B_i)$ and $U^{\mathrm{in}}_{\mathrm{bu}}(B_i) = P^{\mathrm{in}}_{\mathrm{bu}}(B_i)\, u^{\mathrm{in}}_{\mathrm{bu}}(B_i)$. Once we substitute these expressions into (14), we obtain a pair of linear equations for $u^{\mathrm{in}}_{\mathrm{se}}(B_i)$ and $u^{\mathrm{in}}_{\mathrm{bu}}(B_i)$. Solving these equations yields the following buyer and seller payoffs:[44]

$$\begin{aligned} u^{\mathrm{in}}_{\mathrm{se}}(B_i) &= g^{\mathrm{in}} \frac{\delta\, P^{\mathrm{in}}_{\mathrm{bu}}(B_i) - 1}{\delta\, P^{\mathrm{in}}_{\mathrm{bu}}(B_i) + \delta\, P^{\mathrm{in}}_{\mathrm{se}}(B_i) - 2}; \\ u^{\mathrm{in}}_{\mathrm{bu}}(B_i) &= g^{\mathrm{in}} \frac{\delta\, P^{\mathrm{in}}_{\mathrm{se}}(B_i) - 1}{\delta\, P^{\mathrm{in}}_{\mathrm{bu}}(B_i) + \delta\, P^{\mathrm{in}}_{\mathrm{se}}(B_i) - 2}. \end{aligned} \tag{15}$$

4.3 Lump-sum Costs in Formal and Informal Markets

As discussed in Section 3.2.1, $T^{\mathrm{fo}}_{\mathrm{se}}$ is the effective tax rate paid by sellers in the formal market. As in Prado (2011), in this section we also consider other per-period costs that are incurred by a seller who participates in these markets. In particular, a seller participating in the formal market must incur lump-sum costs (independent of profits) which we will denote by $L^{\mathrm{fo}}_{\mathrm{se}}$. These lump-sum costs involve the combined cost of having retail space and paying for licensing fees to comply with government regulations (e.g. fire safety). Likewise, formal buyers incur transportation costs that are independent of whether they acquire the product or not. These lump-sum costs are represented by $L^{\mathrm{fo}}_{\mathrm{bu}}$.

When sellers operate in the informal market, they face an implicit "tax" rate $T^{\mathrm{in}}_{\mathrm{se}}$ which reflects the risks of theft and confiscation. Informal sellers may also have to pay bribes to corrupt police officials or "protection fees" to organized crime against confiscation and theft risks; these are lump-sum payments, independent of a seller's earnings.[45] Thus, we further assume that each informal seller also incurs a lump-sum cost of $L^{\mathrm{in}}_{\mathrm{se}}$ dollars. Informal buyers incur an opportunity

[43] Recall that the maximizer of the Nash trading protocols is the midpoint of the diagonal line from $(g^{\mathrm{in}}, 0)$ to $(0, g^{\mathrm{in}})$

[44] If $\delta = 1$, then the bargaining outcome (15) can be seen as a particular case of the abstract surplus-division model considered in Section 3.3.2. To see this, let $\eta(B_i) := u^{\mathrm{in}}_{\mathrm{se}}(B_i)/g^{\mathrm{in}}$, where $u^{\mathrm{in}}_{\mathrm{se}}(B_i)$ is defined as in Eq.(15). Then η satisfies the conditions proposed in Section 3: it is an increasing function of B_i (because $u^{\mathrm{in}}_{\mathrm{se}}$ is a decreasing function of B_i, and the limit (8) holds because $u^{\mathrm{in}}_{\mathrm{se}}(0) = g^{\mathrm{in}}$.

[45] We refer to the reader to Putnins and Sauka (2015) and Chmielowski (2015) for more details.

cost of waiting around for sellers to arrive; we represent this by a lump-sum cost of $L^{\text{fo}}_{\text{bu}}$ dollars.

Let us define $L_{\text{se}} := L^{\text{fo}}_{\text{se}} - L^{\text{in}}_{\text{se}}$ ($L_{\text{bu}} := L^{\text{fo}}_{\text{bu}} - L^{\text{in}}_{\text{bu}}$) which represents the *net* lump-sum cost for sellers (buyers) in the formal sector. (This could be positive or negative, depending on whether the costs in the informal sector are lower or higher than those in the formal sector.) For modelling purposes, it is equivalent to suppose that informal buyers and sellers face *no* lump sum costs, whereas formal buyers and sellers face lump sum costs of L_{bu} and L_{se} respectively.

Let $R^{\text{fo}}_{\text{se}} := 1 - T^{\text{fo}}_{\text{se}}$ ($R^{\text{in}}_{\text{se}} := 1 - T^{\text{in}}_{\text{se}}$) denote the "residual" earnings rate of sellers in the formal (informal) markets after proportional taxes are paid. Let $R_{\text{se}} := R^{\text{fo}}_{\text{se}}/R^{\text{in}}_{\text{se}}$; this is effectively the "net" residual earnings rate for formal sellers, if we normalize the informal sellers' residual earnings rate to 1. Note that this is equivalent to assuming that informal sellers capture *all* their earnings, while formal sellers only capture a proportion R_{se}. This can represent a situation where informal sellers face *no* risk of theft, while formal sellers pay an *effective* tax rate of $T_{ax} := 1 - R_{\text{se}}$. Note that if expected losses due to theft in the informal market are higher that the formal tax rate, then we will have $R_{\text{se}} > 1$, which implies that $T_{ax} < 0$.

4.4 Dynamic Equilibrium

Having specified all differential costs of trading in formal and informal markets, we can now analyze the corresponding dynamic equilibrium for this new environment. As in Section 3, we consider the dynamic equilibrium induced primarily by best response dynamics. This yields the following dynamic equations:

$$\begin{array}{rcccl} \dot{s}^{\text{fo}}(t) & = & -\dot{s}^{\text{in}}(t) & = & \lambda_{\text{se}}\left((1-T_{ax})\,U^{\text{fo}}_{\text{se}}\left(\dfrac{b^{\text{fo}}(t)}{s^{\text{fo}}(t)}\right) - L_{\text{se}} - U^{\text{in}}_{\text{se}}\left(\dfrac{b^{\text{in}}(t)}{s^{\text{in}}(t)}\right)\right) \\ \dot{b}^{\text{fo}}(t) & = & -\dot{b}^{\text{in}}(t) & = & \lambda_{\text{bu}}\left(U^{\text{fo}}_{\text{bu}}\left(\dfrac{b^{\text{fo}}(t)}{s^{\text{fo}}(t)}\right) - L_{\text{bu}} - U^{\text{in}}_{\text{bu}}\left(\dfrac{b^{\text{in}}(t)}{s^{\text{in}}(t)}\right)\right); \end{array} \tag{16}$$

where $\dot{b}^{\text{fo}}(t)$, $\dot{b}^{\text{in}}(t)$, $\dot{s}^{\text{fo}}(t)$, and $\dot{s}^{\text{in}}(t)$ represent the corresponding time derivatives and $\lambda_{\text{bu}} : \mathbb{R}\longrightarrow\mathbb{R}$ and $\lambda_{\text{se}} : \mathbb{R}\longrightarrow\mathbb{R}$ are strictly increasing functions that modulate the speed of adjustment, where $\lambda_{\text{bu}}(0) = 0 = \lambda_{\text{se}}(0)$.

As in Section 3, a seller finds the formal market more attractive than the informal market if and only if $(1-T_{ax})\,U^{\text{fo}}_{\text{se}}(b^{\text{fo}}/s^{\text{fo}}) - L_{\text{se}} > U^{\text{in}}_{\text{se}}(s^{\text{in}}/b^{\text{in}})$. Likewise, a buyer prefers the formal market if and only if $U^{\text{fo}}_{\text{bu}}(b^{\text{fo}}/\frac{b^{\text{in}}}{s^{\text{in}}}s^{\text{fo}}) - L_{\text{bu}} > U^{\text{in}}_{\text{bu}}(s^{\text{in}}/b^{\text{in}})$. As a result, the necessary and sufficient condition for a population distribution $(b^{\text{fo}}, b^{\text{in}}, s^{\text{fo}}, s^{\text{in}})$ to be an equilibrium is that these shares are a fixed point of equation (16), which requires that

$$(1-T_{ax})\,U^{\text{fo}}_{\text{se}}\left(\frac{b^{\text{fo}}}{s^{\text{fo}}}\right) - L_{\text{se}} = U^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right) \quad \text{and} \quad U^{\text{fo}}_{\text{bu}}\left(\frac{b^{\text{fo}}}{s^{\text{fo}}}\right) - L_{\text{bu}} = U^{\text{in}}_{\text{bu}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right). \tag{17}$$

Since $b^{\text{fo}} + b^{\text{in}} = b$ and $s^{\text{fo}} + s^{\text{in}} = 1$, the market is completely described by the ordered pair

$(b^{\text{in}}, s^{\text{in}})$, and equation (17) which reduces to:

$$(1 - T_{ax})\, U^{\text{fo}}_{\text{se}}\left(\frac{b - b^{\text{in}}}{1 - s^{\text{in}}}\right) - L_{\text{se}} \;=\; U^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right) \text{ and } U^{\text{fo}}_{\text{bu}}\left(\frac{b - b^{\text{in}}}{1 - s^{\text{in}}}\right) - L_{\text{bu}} \;=\; U^{\text{in}}_{\text{bu}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right). \quad (18)$$

An equilibrium (b^*, s^*) is *locally stable* if there exists some neighbourhood $\mathcal{U}$ around (b^*, s^*) such that, for any $(b^{\text{in}}, s^{\text{in}}) \in \mathcal{U}$, the forward-time orbit of $(b^{\text{in}}, s^{\text{in}})$ under (16) converges to (b^*, s^*). Graphically, it is easy to identify a locally stable equilibrium. To this end, let us rewrite (16) more generally as follows:

$$\dot{b}^{\text{in}} = \beta(b^{\text{in}}, s^{\text{in}}) \qquad \text{and} \qquad \dot{s}^{\text{in}} = \sigma(b^{\text{in}}, s^{\text{in}}).$$

Here, β and σ are the functions appearing on the right hand side of equation (16). Then an equilibrium is simply an intersection of the two *isoclines* $\mathcal{B} := \{(b^{\text{in}}, s^{\text{in}});\;\; \beta(b^{\text{in}}, s^{\text{in}}) = 0\}$ and $\mathcal{S} := \{(b^{\text{in}}, s^{\text{in}});\; \sigma(b^{\text{in}}, s^{\text{in}}) = 0\}$. Typically, $\mathcal{B}$ and $\mathcal{S}$ are smooth curves in the rectangular domain $[0, b] \times [0, 1]$ and are given by:

$$\begin{aligned} \mathcal{S}(T_{ax}, L_{\text{se}}) &:= \left\{(s^{\text{in}}, b^{\text{in}}) \in [0,1] \times [0,b]\;;\; (1 - T_{ax})\, U^{\text{fo}}_{\text{se}}\left(\frac{b - b^{\text{in}}}{1 - s^{\text{in}}}\right) - L_{\text{se}} = U^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right)\right\}, \\ \mathcal{B}(L_{\text{bu}}) &:= \left\{(s^{\text{in}}, b^{\text{in}}) \in [0,1] \times [0,b]\;;\; U^{\text{fo}}_{\text{bu}}\left(\frac{b - b^{\text{in}}}{1 - s^{\text{in}}}\right) - L_{\text{bu}} = U^{\text{in}}_{\text{bu}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right)\right\}. \end{aligned} \quad (19)$$

The equilibrium (b^*, s^*) is locally stable if the following conditions are met in a neighbourhood of (b^*, s^*):

(i) The absolute slope of $\mathcal{B}$ at (b^*, s^*) is larger than the absolute slope of $\mathcal{S}$ at this point.[46]

(ii) β is positive to the left of $\mathcal{B}$, and negative to the right of $\mathcal{B}$.

(iii) σ is positive below $\mathcal{S}$, and negative above $\mathcal{S}$.

If the population of informal buyers unilaterally dips below (above) b^*, then Condition (ii) says that the payoff for informal buyers will be higher (lower) than the payoff for formal buyers, causing buyers to migrate into (out of) the informal market until $b^{\text{in}} = b^*$. Likewise, if the population of informal sellers unilaterally dips below (above) s^*, then Condition (iii) says that the payoff for informal sellers will be higher (lower) than the payoff for formal sellers, causing sellers to migrate into (out of) the informal market, until $s^{\text{in}} = s^*$. Thus, a stable equilibrium is such that any point to the left (right) of $\mathcal{B}$ will move in a rightward (leftwards) direction and any point below (above) $\mathcal{S}$ will move upwards (downwards).

[46] Heuristically, this means we can think of $\mathcal{B}$ as a roughly "vertical" curve near (b^*, s^*), whereas $\mathcal{S}$ is roughly "horizontal" near (b^*, s^*).

We say there is a *pure formal market equilibrium* if the point $(b^{\text{in}}, s^{\text{in}}) = (0,0)$ satisfies the equilibrium condition given by (18). We say there is a *pure informal market equilibrium* if the point $(b^{\text{in}}, s^{\text{in}}) = (b,1)$ satisfies equation (18). Finally a *mixed-market equilibrium* is a point $(b^*, s^*) \in (0,b)\times(0,1)$ which satisfies equation (18). To establish the robust co-existence of formal and informal markets, we must show that *there exists a locally stable mixed-market equilibrium.*

Remark 2. The equilibrium equation (18) can also be interpreted as an uncorrelated, symmetric mixed-strategy Nash equilibrium or an equilibrium under replicator/imitation dynamics; see Appendix B for details.

4.5 Properties of the equilibrium

Given the complexity of the model, no simple closed form solutions exist, a numerical analysis is required to determine further properties of the equilibrium. We now examine different scenarios and explore the corresponding equilibrium properties.

4.5.1 No Taxes and No Quality Assurance

To isolate the implications of the trading protocol, we first consider an environment with $T_{ax} = 0$ and $g^{\text{in}} = g^{\text{fo}}$. In other words, we initially suppose that the formal market has no quality assurance advantage, and that neither market has a tax advantage. This would occur, for example, if the tax rate in the formal market exactly matched the rate of theft in the informal market, and if products had zero probability of defects or if $\alpha(q) = q$ for all q.

When $T_{ax} = 0$ and $L_{\text{bu}} = L_{\text{se}} = 0$, the two curves $\mathcal{S}(0,0)$ and $\mathcal{B}(0)$ characterizing the stability of the equilibrium are very close to the diagonal. Heuristically, this means that buyers and sellers are both essentially indifferent between the two markets, as long as

$$\frac{b^{\text{in}}}{s^{\text{in}}} \;=\; b \;=\; \frac{b^{\text{fo}}}{s^{\text{fo}}}. \tag{20}$$

Numerical analysis suggest that, in this case, buyers and sellers exhibit a very weak preference for an all-formal market equilibrium. But the difference in payoff between the all-formal market equilibrium and other points on the diagonal (20) is so small that all points on this diagonal could be regarded as "quasi-equilibria". However, if $L_{\text{bu}} \neq 0$ and $L_{\text{se}} \neq 0$, then the picture becomes much clearer.

Result 2. *Suppose $T_{ax} = 0$. Then numerical methods suggest that:*

(a) *if L_{bu} and L_{se} have opposite signs, and $|L_{\text{bu}}|$ and $|L_{\text{se}}|$ are large enough, then there is a locally stable mixed-market equilibrium;*

(b) *if* $L_{\text{bu}} < 0$ *and* $L_{\text{se}} < 0$*, and* $|L_{\text{bu}}|$ *and* $|L_{\text{se}}|$ *are large enough, there exists a locally stable pure formal market equilibrium;*

(c) *if* $L_{\text{bu}} > 0$ *and* $L_{\text{se}} > 0$*, and* $|L_{\text{bu}}|$ *and* $|L_{\text{se}}|$ *are large enough, there exists a locally stable pure informal market equilibrium.*

In all three cases, the equilibrium appears to be unique.

To gain some deeper understanding of Result 2, notice that an equilibrium (18) is any crossing point of the isocline $\mathcal{B}(L_{\text{bu}})$ (from equation (19)) and the isocline

$$\mathcal{S}(0, L_{\text{se}}) \quad := \quad \left\{(s^{\text{in}}, b^{\text{in}}) \in [0,1] \times [0,b]\ ;\ U^{\text{fo}}_{\text{se}}\left(\frac{b - b^{\text{in}}}{1 - s^{\text{in}}}\right) - L_{\text{se}} = U^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right)\right\}.$$

Let us now define the functions $\beta, \sigma : [0,b] \times [0,1] \longrightarrow \mathbb{R}$ by setting

$$\begin{aligned} \sigma(b^{\text{in}}, s^{\text{in}}) &:= U^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right) - U^{\text{fo}}_{\text{se}}\left(\frac{b - b^{\text{in}}}{1 - s^{\text{in}}}\right) \\ \text{and} \qquad \beta(b^{\text{in}}, s^{\text{in}}) &:= U^{\text{in}}_{\text{bu}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right) - U^{\text{fo}}_{\text{bu}}\left(\frac{b - b^{\text{in}}}{1 - s^{\text{in}}}\right), \end{aligned}$$

for all $b^{\text{in}} \in [0,b]$ and $s^{\text{in}} \in [0,1]$. Suppose $L_{\text{se}} = 0$; then σ measures how relatively attractive the informal market is for sellers. If $\sigma(b^{\text{in}}, s^{\text{in}})$ is positive (negative), then sellers will move into (out of) the informal market, so s^{in} will increase (decrease). Likewise, suppose $L_{\text{bu}} = 0$; then β measures how relatively attractive the informal market is for buyers. If $\beta(b^{\text{in}}, s^{\text{in}})$ is positive (negative), then buyers will move into (out of) the informal market, so b^{in} will increase (decrease). The isocontours of β are the isoclines $\mathcal{B}(L_{\text{bu}})$ for various choices of L_{bu}. The isocontours of σ are the isoclines $\mathcal{S}(0, L_{\text{se}})$ for various choices of L_{se}. These isocontours cross if and only if the gradient vector field $\nabla\sigma$ is *not* parallel to the gradient vector field $\nabla\beta$. So this is what we must verify to demonstrate Result 2.

If the two gradient vector fields *were* parallel, then we would have

$$\phi(b^{\text{in}}, s^{\text{in}}) \quad := \quad \frac{\nabla\sigma(b^{\text{in}}, s^{\text{in}}) \bullet \nabla\beta(b^{\text{in}}, s^{\text{in}})}{\|\nabla\sigma(b^{\text{in}}, s^{\text{in}})\| \cdot \|\nabla\beta(b^{\text{in}}, s^{\text{in}})\|} \quad = \quad \pm 1, \tag{21}$$

for all $b^{\text{in}} \in [0,b]$ and $s^{\text{in}} \in [0,1]$. Using a symbolic computation package like Mathematica or Maple, it is easy to verify that $\phi(b^{\text{in}}, s^{\text{in}}) \neq \pm 1$, for any choice of $(b^{\text{in}}, s^{\text{in}})$ which is not close to the diagonal line $\{(b^{\text{in}}, s^{\text{in}});\ b^{\text{in}}/s^{\text{in}} = b/s\}$ or the lines $s^{\text{in}} = 0$ or $s^{\text{in}} = 1$.[47]

Any crossing of the isoclines $\mathcal{B}(L_{\text{bu}})$ and $\mathcal{S}(0, L_{\text{se}})$ will determine an equilibrium (18) of the economy. However, not all such equilibria are locally stable. If the absolute slope of $\mathcal{S}(0, L_{\text{se}})$ is

[47] `Maple` source code for this computation and all the computations described in the rest of this section is available on request from the authors.

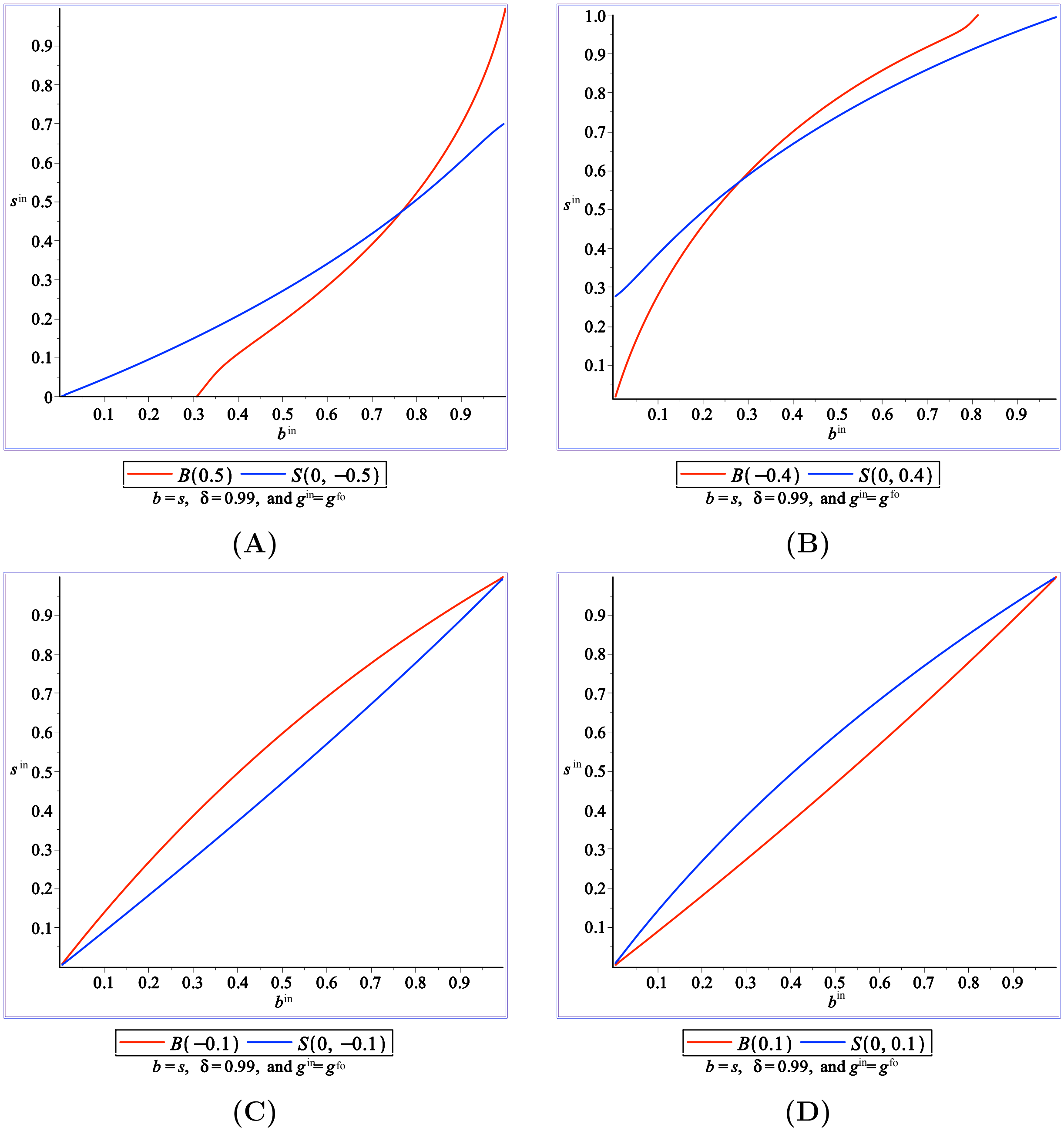


Figure 1: **(A)** A locally stable mixed-market equilibrium defined by the crossing of $\mathcal{S}(0,-0.5)$ (dashed line) and $\mathcal{B}(0.5)$ (solid line). **(B)** A locally stable mixed-market equilibrium defined by the crossing of $\mathcal{S}(0,0.4)$ and $\mathcal{B}(-0.4)$. **(C)** A pure formal market equilibrium $(0,0)$ exists for $\mathcal{S}(0,-0.1)$ and $\mathcal{B}(-0.1)$. **(D)** A pure informal market equilibrium (b,s) for $\mathcal{S}(0,0.1)$ and $\mathcal{B}(0.1)$.

less than the absolute slope of $\mathcal{B}(L_{bu})$ when they cross, then it is easy to check that conditions (i)-(iii) from Section 4.4 are satisfied, so that the equilibrium is locally stable. For illustration purposes of Result 2, consider $b = s$, $\delta = 0.99$ and $g^{in} = g^{fo}$; the $\mathcal{S}$-isoclines are the dashed curves, while the $\mathcal{B}$-isoclines are the solid curves. In particular, Figure 1(A) shows the curves $\mathcal{B}(0.5)$ and $\mathcal{S}(0, -0.5)$ intersecting in a locally stable equilibrium. Figure 1(B) shows the curves $\mathcal{B}(-0.4)$ and $\mathcal{S}(0, 0.4)$ intersecting in a locally stable equilibrium. Furthermore, a plot of the vector fields defined by best response differential equations (16) reveals that these equilibria are in fact *global* attractors; see Figures 5(A,B) in Appendix D.[48]

Thus, there is a stable equilibrium with a mixture of formal and informal markets whenever the buyers and sellers face lump-sum costs in different markets. However, if both buyers and sellers face lump-sum costs in the *same* market, then the isoclines do *not* cross. In this case, the dynamics cause all buyers and sellers to migrate to the market *without* the lump-sum costs. If $\mathcal{S}(0, L_{se})$ is always below $\mathcal{B}(L_{bu})$, then all buyers and sellers migrate to the formal market, as described by Result 2(b). If $\mathcal{S}(0, L_{se})$ is always above $\mathcal{B}(L_{bu})$, then all buyers and sellers migrate to the informal market, as described by Result 2(c). Figures 1(C,D) illustrate these cases. (Once again, the associated vector fields reveals that these equilibria are global attractors; see Figures 5(C,D) in Appendix D.)

4.5.2 Crime and taxation

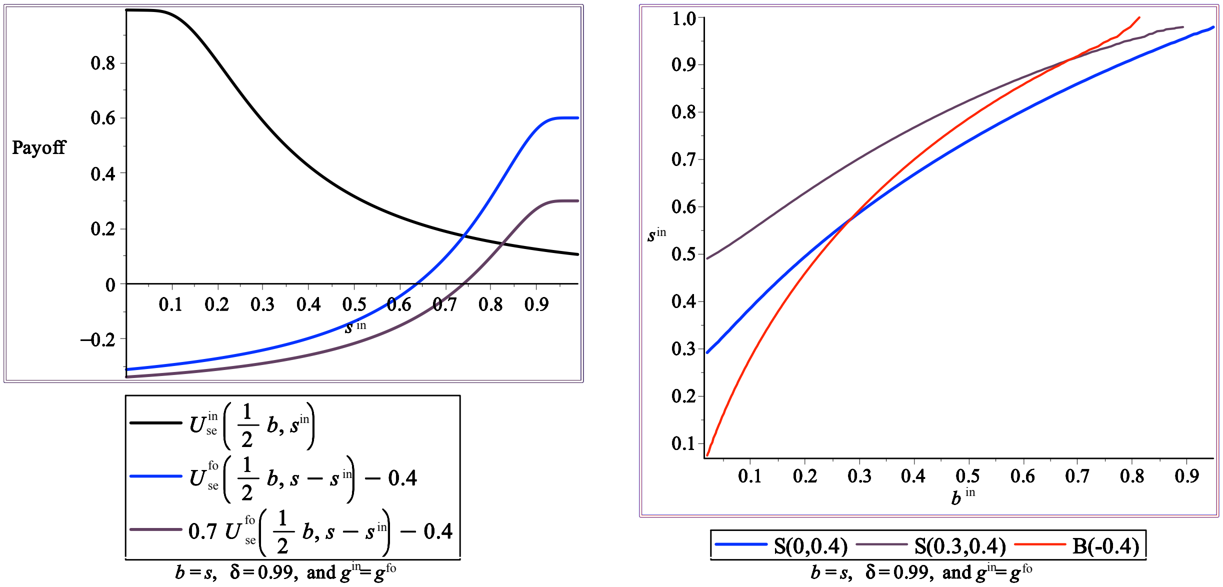
In the previous numerical example, we studied the case where the tax rate in the formal sector is exactly equal to the crime rate in the informal sector, so that $T_{ax} = 0$. *Ceteris paribus*, raising tax rates in the formal sector (or lowering crime rates in the informal sector) will cause some buyers and sellers to migrate from the formal to the informal market. To see this, consider the isocline

$$\mathcal{S}(T_{ax}, L_{se}) \quad := \quad \left\{ (s^{in}, b^{in}) \in [0,1] \times [0,b] \; ; \; (1 - T_{ax})\, U_{se}^{fo}\left(\frac{b - b^{in}}{1 - s^{in}}\right) - L_{se} = U_{se}^{in}\left(\frac{b^{in}}{s^{in}}\right) \right\}$$

for any net tax level T_{ax}. We claim that increasing T_{ax} will cause this curve to shift upwards. As a result, a higher formal taxes will cause a larger fraction of both buyers and sellers to migrate to the informal sector. However, as long as the net tax is small enough, the equilibrium is such that formal and informal markets of non-trivial size exists.

To illustrate this point, suppose for simplicity that $L_{se} = 0.4$ and $b^{in} = b/2$. In Figure 2(a), the horizontal axis represents s^{in}, and the downward sloping curve is $U_{se}^{in}(b/2, s^{in})$ —the payoff for informal sellers, as a function of s^{in}. The upward sloping curves are the payoffs for formal sellers, as a function of s^{in}. The dotted curve is $U_{se}^{fo}(b/2, s - s^{in}) - 0.4$; this is the payoff with $T_{ax} = 0$ (i.e. no net taxation). The dashed curve is $0.7 \cdot U_{se}^{fo}(b/2, s - s^{in}) - 0.4$; this is the payoff

[48] We find the same qualitative results when alternative parameterizations are used.



(a) (b)

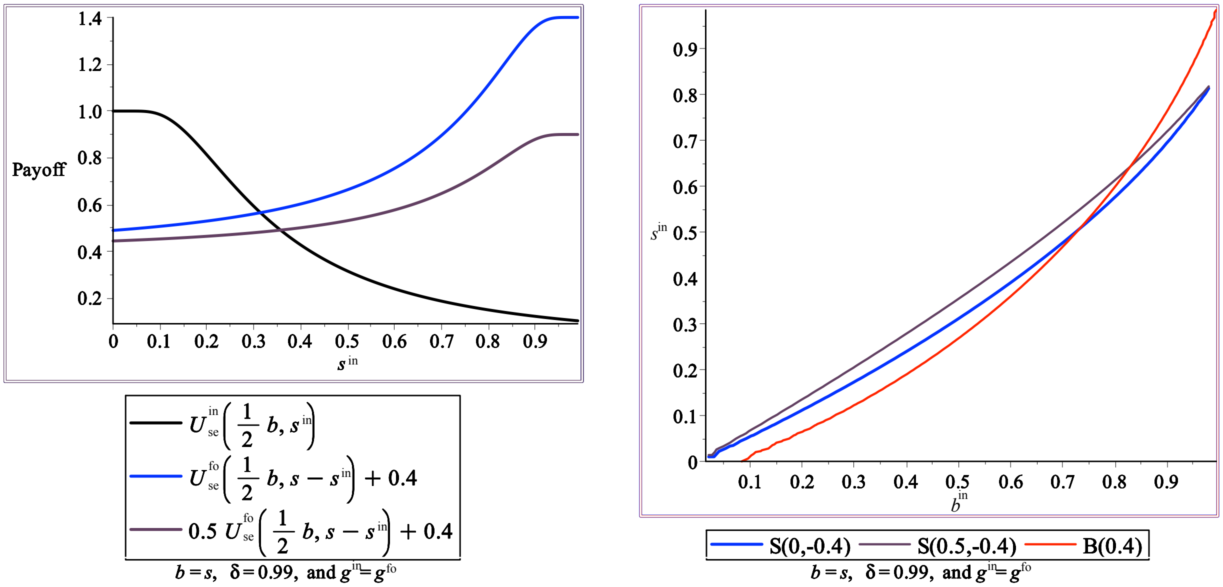
Figure 2: Crime and taxation when $L_{se} > 0 > L_{bu}$.

with $\mathcal{T}_{ax} = 0.3$ (i.e. a net taxation rate of 30%). Note how the intersection with $U^{in}_{se}(b/2, s^{in})$ shifts to the right as we increase $\mathcal{T}_{ax}$, indicating that equilibrium occurs at a higher value of s^{in} (i.e. more sellers enter the informal market). By repeating this argument for every value of b^{in}, we can see that the curve $\mathcal{S}(0.3, 0.4)$ must be *above* the curve $\mathcal{S}(0, 0.4)$. Since $\mathcal{B}$ slopes upwards, the intersection of $\mathcal{S}(0.3, 0.4)$ with the curve $\mathcal{B}(L_{bu})$ will thus be *northeast* of the intersection of $\mathcal{S}(0, 0.4)$ with the curve $\mathcal{B}(L_{bu})$, as shown in Figure 2(b). In other words, higher formal taxes will cause a larger fraction of both buyers and sellers to migrate to the informal sector. However, as long as the net tax is small enough, the new equilibrium is still a mixed-market type. Figure 2 illustrates the claim for $L_{se} := 0.4$ and $L_{bu} = -0.4$, but we would get a similar picture for any $L_{bu} < 0 < L_{se}$.[49]

Figure 2 showed the case when $L_{se} > 0 > L_{bu}$ so that sellers must pay a net lump-sum cost to enter the formal sector (e.g. the costs of retail space and licenses), while buyers pay a net lump-sum cost to enter the formal sector (e.g. inconvenience). Figure 3 shows the opposite case, when $L_{se} < 0 < L_{bu}$. In particular, we set $L_{se} = -0.4$ while $L_{bu} := 0.4$, and we compare the net tax levels $\mathcal{T}_{ax} = 0$ and $\mathcal{T}_{ax} = 0.5$. The impact of taxation is similar to that in Figure 2.[50]

Note that the impact of taxation is stronger in Figure 2 than in Figure 3, despite the fact that the net tax increase in Figure 3 was $\mathcal{T}_{ax} = 0.5$, whereas in Figure 2 it was only $\mathcal{T}_{ax} = 0.3$. In other words, the effect of taxation depends on the relative lump-sum entry costs sellers and buyers face in the formal market: taxation causes a stronger effect in a situation when the sellers

[49] We find the same qualitative results when alternative parameterizations are used.
[50] We would get a similar picture for any $L_{bu} > 0 > L_{se}$.



(a) (b)

Figure 3: Crime and taxation when $L_{se} < 0 < L_{bu}$.

(but not buyers) must pay a net positive fee to enter the formal market, whereas taxation causes a weaker effect when it is buyers (but not sellers) who must pay a net fee to enter the formal market.[51]

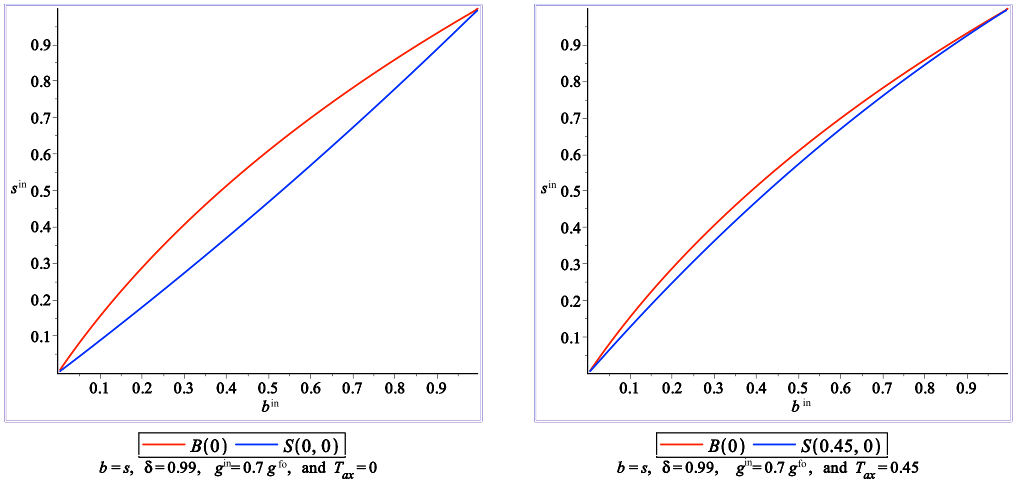
The results in Sections 4.5.1 and 4.5.2 are consistent with the work of Oviedo et al. (2009), among other authors, who document the types of policies that high-income OECD countries have used to redirect workers from the informal economy into the formal market. The most common policies or reform packages used by various countries tend to include five crucial elements: (i) reducing the costs of operating in the formal market; (ii) improving the audit technology and enforcement in the informal economy; (iii) improving the taxing authority's communication strategy with the formal sector; (iv) modernizing administrative and regulatory processes and functions in the formal market and (v) providing basic social protection for all, so that the informal economy would not be viewed by the unemployed as a substitute for social protection.

4.5.3 Quality assurance versus taxation

In all of our numerical analysis so far, we have assumed that $g^{in} = g^{fo}$, so that the formal market has no advantage over the informal market due to quality assurance. This would be the case, for example, if the quality assurance technology has constant returns to scale.

In this section, we examine a situation where $g^{in} < g^{fo}$ so that quality assurance gives the formal market an advantage and creates an extra surplus that the government can tax. For instance, let us suppose that $g^{in} = 0.7\, g^{fo}$. Figure 4(a) shows a market with no net taxation and

[51] We find the same qualitative results when alternative parameterizations are used.



(a) (b)

Figure 4: Quality assurance versus taxation. (a) If $g^{\text{in}} = 0.7\, g^{\text{fo}}$, then all buyers and sellers migrate to the formal market because of its quality assurance advantage. (b) The formal market remains the only stable equilibrium, even if the government imposes 45% taxation.

no lump-sum costs (i.e. $T_{ax} = L_{\text{se}} = L_{\text{bu}} = 0$). We see that $\mathcal{S}(0,0)$ is always below $\mathcal{B}(0)$, so all buyers and sellers migrate to the formal market. Figure 4(b) shows a market with no lump-sum costs (i.e. $L_{\text{se}} = L_{\text{bu}} = 0$), but with a 45% net tax rate on the formal sector (i.e. $T_{ax} = 0.45$), we see that $\mathcal{S}(0.45, 0)$ is *still* below $\mathcal{B}(0)$, so that all buyers and sellers remain in the formal market. Thus, if the formal market has even a small quality assurance advantage, then it can withstand a large amount of government taxation.

Notwithstanding the absence of an informal market in the equilibrium in Figure 4, if the burden of taxation or regulation in the formal market is high enough, then the equilibrium will involve a nonzero amount of informal activity, despite the formal market's quality assurance advantage. Thus, in contrast to the conventional wisdom and the literature cited above, our analysis suggests that introducing *more* regulation can sometimes shift activity from informal to formal markets. Of course, this will only occur if the advantages of enhanced quality assurance outweigh the administrative costs of regulatory compliance. Note that our model predictions are consistent with the findings of Gambetta (1988), Beckert (2005) and Mollering (2006). These authors highlight that when purchasing goods from the informal sector, risks faced by informal buyers arise from the asymmetric information regarding product quality in light of incomplete or non-enforceable contracts when buying in these markets.[52]

[52] We refer to Loayza (1996) for the role of confiscation in the informal sector.

5 Conclusion

There is a long tradition in economics of investigating which trading protocols survive in equilibrium. In this paper, we explore when bargaining and price posting can co-exist. We do so within the context of the formal and informal markets, as these two trading protocols have very different informational requirements for trade to take place. In particular, formal sellers must publicly advertise their prices and locations in order to attract buyers. Price posting is then a suitable trading protocol for formal sellers. Instead, informal sellers *avoid* publicly disclosing their exact locations and prices as to evade taxes and regulations. Bargaining then fits the needs of informal sellers. Building on this insight, we also consider other distinguishing features (taxes, crime and quality assurance) to determine when price posting and bargaining can co-exist. To this end, we consider an evolutionary framework where agents' payoffs depend on the ratio of buyers and sellers in each of these markets. All agents try to position themselves in the market which can yield them the highest possible payoff. This strategic interaction in turn critically affects the relative size and the evolution of these two markets.

When only sellers can switch between formal and informal markets, we analytically show that formal and informal markets of nontrivial size co-exist in a stable equilibrium. We also demonstrate that some sellers will switch from formal to informal markets whenever the formal sellers' quality assurance erodes, the government imposes higher taxes and regulations in the formal market, the risk of crime and/or confiscation decreases in the informal market, or the number of buyers in the informal market increases. Conversely, sellers will switch from the informal to the formal market whenever the contrary changes take place. The contrast between our conclusions and those of Lu and McAfee (1996) and Kultti (1999) is partly because agents in our environment are replaced differently, and partly because we also consider various additional features that distinguish the formal and informal markets. These extra asymmetries affect the differential seller's payoffs when trading in formal and informal markets and determining when co-existence is possible.

If we relax the immobility of buyers and allow both buyers and sellers to switch between formal and informal markets, and if the net lump-sum cost for a seller in the formal sector relative to that in the informal sector and the net lump-sum cost for a buyer in the formal sector relative to that in the informal sector have opposite signs, then we again obtain a locally stable equilibrium in which formal and informal markets co-exist. However, if the above-mentioned different relative net lump-sum costs are both negative (positive), then there exist a unique locally stable equilibrium where only formal (informal) activity takes place in the long run.

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Appendix A: Proofs

Proof of Lemma A. The efficient value q^* of investment in quality assurance is the value such that $\alpha'^*) = 1$ (i.e. such that one additional cent spent on quality assurance increases the buyer's expected utility by exactly one cent). Since α' is nonincreasing (by concavity), we have $\alpha'(q) \geq 1$ for all $q \in [0, q^*]$. Thus, since $\alpha(0) = 0$, the Fundamental Theorem of Calculus implies that $\alpha(q^*) \geq q^*$ (i.e. the benefit of quality assurance outweighs its cost). Thus,

$$\frac{g^{\text{fo}}}{g^{\text{in}}} \quad = \quad \frac{v_{\text{bu}}^{\text{fo}} - c_{\text{se}}^{\text{fo}}}{v_{\text{bu}}^{\text{in}} - c_{\text{se}}^{\text{in}}} \quad = \quad \frac{v_{\text{bu}}^{\text{in}} + \alpha(q) - c_{\text{se}}^{\text{in}} - q}{v_{\text{bu}}^{\text{in}} - c_{\text{se}}^{\text{in}}} \quad = \quad 1 + \frac{\alpha(q) - q}{v_{\text{bu}}^{\text{in}} - c_{\text{se}}^{\text{in}}} \quad \geq \quad 1,$$

because $\alpha(q) \geq q$. Thus, $g^{\text{in}} \leq g^{\text{fo}}$. □

Proof of Theorem 1. We must show that the interval $[0, 1]$ contains a zero for the function $\widetilde{U}$. By inspecting formulae (5) and (9), we see that, for all $s^{\text{fo}} \in [0, 1]$, we have

$$\begin{aligned} \widetilde{U}(s^{\text{fo}}) = \widetilde{U}_{\text{se}}^{\text{fo}}(s^{\text{fo}}) - \widetilde{U}_{\text{se}}^{\text{in}}(s^{\text{fo}}) \quad &= \quad (1 - T_{\text{se}}^{\text{fo}})\, \widehat{U}(s^{\text{fo}}), \\ \text{with} \quad \widehat{U}(s^{\text{fo}}) &:= \widehat{U}_1(s^{\text{fo}}) - K\, \widehat{U}_2(s^{\text{fo}}), \\ \text{where} \quad K &:= \frac{(1 - T_{\text{se}}^{\text{in}})\, g^{\text{in}}}{(1 - T_{\text{se}}^{\text{fo}})}, \\ \text{while} \quad \widehat{U}_1(s^{\text{fo}}) &:= \left[1 - \exp\left(\frac{-b^{\text{fo}}}{s^{\text{fo}}}\right)\right] \cdot \left(1 - \frac{b^{\text{fo}}/s^{\text{fo}}}{\exp(b^{\text{fo}}/s^{\text{fo}}) - 1}\right), \quad \text{by Eq.(5)}, \\ \text{and} \quad \widehat{U}_2(s^{\text{fo}}) &:= \eta\left(\frac{b^{\text{in}}}{1 - s^{\text{fo}}}\right) \cdot \left[1 - \exp\left(\frac{-b^{\text{in}}}{1 - s^{\text{fo}}}\right)\right] \qquad \text{by Eq.(9).} \end{aligned} \tag{22}$$

Clearly, it will be sufficient to find a zero for $\widehat{U}$ instead. From equation (8), simple computations yield:

$$\lim_{s \searrow 0} \widehat{U}(s) \;=\; 1 - K\, \eta(b^{\text{in}})\, (1 - \exp(-b^{\text{in}})) \quad \text{and} \quad \lim_{s \nearrow 1} \widehat{U}(s) \;=\; 1 - \frac{1 + b^{\text{fo}}}{\exp(b^{\text{fo}})} - K.$$

From here, it is easy to check that

$$\begin{aligned} \left(K < \frac{1}{\eta(b^{\text{in}})\, (1 - \exp(-b^{\text{in}}))}\right) \quad &\Longrightarrow \quad \left(\lim_{s \searrow 0} \widehat{U}(s) > 0\right) \\ \text{and} \quad \left(K > 1 - \frac{b^{\text{fo}} + 1}{\exp(b^{\text{fo}})}\right) \quad &\Longrightarrow \quad \left(\lim_{s \nearrow 1} \widehat{U}(s) < 0\right). \end{aligned} \tag{23}$$

But $\widehat{U}$ is continuous on $[0, 1]$. Thus, if K satisfies both the conditions in (23), then the Intermediate Value Theorem implies that $\widehat{U}(s^*) = 0$ for some $s^* \in (0, 1)$. Furthermore, $\widehat{U}$ is going from positive values (near 0) to negative values (near 1), so $\widehat{U}$ must be decreasing near s^*; hence s^* is a stable equilibrium.

Now, it is easy to check that the function $\widehat{U}_2$ is positive everywhere on $[0,1]$. Thus, if K increases, then the graph of $\widehat{U}$ will move downwards everywhere. Since $\widehat{U}$ is decreasing near s^*, a downwards movement of the graph will cause s^* to move to the left in the interval $[0,1]$. In other words, s^* will decrease when K increases. By inspection of formula (22), K is increasing with g^{in} and $T_{\text{se}}^{\text{fo}}$, while it is decreasing with $T_{\text{se}}^{\text{in}}$. Thus, s^* is *decreasing* with g^{in} and $T_{\text{se}}^{\text{fo}}$, and increasing with $T_{\text{se}}^{\text{in}}$.

Meanwhile, $\widehat{U}_1$ is clearly increasing as a function of b^{fo}, and independent of b^{in}. On the other hand, $\widehat{U}_2$ is independent of b^{fo}, but increasing as a function of b^{in} (because η is an increasing function, by hypothesis). Thus, $\widehat{U}$ is *decreasing* as a function of b^{in} (because K is positive by inspection of formula (22)). Thus, if we increase b^{fo}, then the graph of $\widehat{U}$ is will move upwards (and hence, s^* will move to the right), whereas if we increase b^{in}, then the graph of $\widehat{U}$ will move downwards (hence, s^* will move to the left). Thus, s^* is an increasing function of b^{fo}, and a decreasing function of b^{in}. □

Appendix B: Replicator/Imitation Dynamics

Here we sketch another evolutionary framework that leads to the equilibrium represented by equation (11). As in the model of best response dynamics described in in Section 3.4, we suppose there is an infinite sequence of time periods, with trade occurring in each market during each time period. But instead of migrating between markets in response to higher payoffs, agents learn by *imitating* other agents. The more agents choose a particular strategy, and the better they are doing relative to the average payoff, the more likely it is that other agents will imitate their behavior.

Alternatively, we can interpret the same model in terms of successive generations of agents. During each time period, some agents produce one or more children, and some agents die. Children remain in the same market as their parents.[53] The net reproductive rate (births minus deaths) of each market type is determined by how much the payoff for that market exceeds the population average payoff. To be precise, the population average payoff for sellers at time t is given by:

$$s^{\text{fo}}(t)\, U_{\text{se}}^{\text{fo}}\left[b^{\text{fo}}(t)/s^{\text{fo}}(t)\right] + s^{\text{in}}(t)\, U_{\text{se}}^{\text{in}}\left[b^{\text{in}}(t)/s^{\text{in}}(t)\right]$$

so the reproductive rate of the *formal* sellers will be:

$$\rho(t) = (1 - s^{\text{fo}}(t))\, U_{\text{se}}^{\text{fo}}\left[b^{\text{fo}}(t)/s^{\text{fo}}(t)\right] - s^{\text{in}}(t)\, U_{\text{se}}^{\text{in}}\left[b^{\text{in}}(t)/s^{\text{in}}(t)\right].$$

[53]Note that, this interpretation is not conducive to view the model as a repeated game, since individual agents only live for one period, after which their offspring replace them.

The population of formal sellers will grow (or shrink) exponentially at this rate. Formally, we have $\dot{s}^{\rm fo}(t) = \lambda_{\rm se}\,\rho(t)\cdot s^{\rm fo}(t)$, where $\lambda_{\rm se} > 0$ is some constant. This leads to the following dynamical equation

$$\dot{s}^{\rm fo}(t) = -\dot{s}^{\rm in}(t) \quad = \quad \lambda_{\rm se}\, s^{\rm fo}(t)\, s^{\rm in}(t)\left(\widetilde{U}^{\rm fo}_{\rm se}\left(\frac{b^{\rm fo}(t)}{s^{\rm fo}(t)}\right) - \widetilde{U}^{\rm in}_{\rm se}\left(\frac{b^{\rm in}(t)}{s^{\rm in}(t)}\right)\right), \tag{24}$$

where $\lambda_{\rm se} > 0$ is a constant. Again, this dynamical equation has both a discrete-time and a continuous-time interpretation. In either case, equation (11) is a necessary and sufficient condition for a population distribution $(s^{\rm fo}, s^{\rm in})$ to be a "nontrivial" fixed point of the dynamics. Here, "nontrivial" refers to the fact that the replicator dynamics always have "trivial" fixed points where $s^{\rm fo} = 0$ or $s^{\rm in} = 0$. However, unless these "pure population" equilibria arise from a solution to equation (11), they are generally unstable to small perturbations. Thus, a pure population of this type will be destabilized as soon as even *one* of the reproducing agents produces a "mutant" child of the opposite type. Thus, we can safely ignore these trivial equilibria, and focus only on the equilibria described by equation (11).

Both Buyers and Sellers Switching between Markets

Finally, we could suppose that the buyer/seller populations *both* evolve according to replicator/imitation dynamics. This yields dynamical equations:

$$\begin{aligned}
\dot{s}^{\rm fo}(t) = -\dot{s}^{\rm in}(t) \quad &= \quad \lambda_{\rm se}\, s^{\rm fo}(t)\, s^{\rm in}(t)\left((1 - T_{ax})\, U^{\rm fo}_{\rm se}\left(\frac{b^{\rm fo}(t)}{s^{\rm fo}(t)}\right) - L_{\rm se} - U^{\rm in}_{\rm se}\left(\frac{b^{\rm in}(t)}{s^{\rm in}(t)}\right)\right) \\
\text{and} & \\
\dot{b}^{\rm fo}(t) = -\dot{b}^{\rm in}(t) \quad &= \quad \lambda_{\rm bu}\, b^{\rm fo}(t)\, b^{\rm in}(t)\left(U^{\rm fo}_{\rm bu}\left(\frac{b^{\rm fo}(t)}{s^{\rm fo}(t)}\right) - L_{\rm bu} - U^{\rm in}_{\rm bu}\left(\frac{b^{\rm in}(t)}{s^{\rm in}(t)}\right)\right),
\end{aligned} \tag{25}$$

where $\lambda_{\rm se} > 0$ and $\lambda_{\rm bu} > 0$ are constants. Again, this dynamical equation has both a discrete-time and a continuous-time interpretation. In either case, equation (18) from Section 4.4 is a necessary and sufficient condition for a population distribution $(b^{\rm fo}, b^{\rm in}, s^{\rm fo}, s^{\rm in})$ to be a "nontrivial" fixed point of the dynamics (25). Here, "nontrivial" refers to the fact that the replicator dynamics always has "trivial" fixed points where either $b^{\rm fo} = 0$ or $b^{\rm in} = 0$ and either $s^{\rm fo} = 0$ or $s^{\rm in} = 0$.

Note that the vector field determined by (25) is obtained by multiplying the vector field defined by (16) by a scalar function which is positive everywhere in $(0, b) \times (0, 1)$. Thus, a stable fixed point for (16) is also a stable fixed point for (25).

Appendix C: Alphabetical Index of Notation

$\alpha(q)$ Benefit (to the formal buyers) of quality assurance (e.g. warranties, free repair service, etc.)

$b^{\rm in}$ Ratio of buyers in the informal market, relative to population of sellers in *both* markets.

b^{fo} Ratio of buyers in the formal market, relative to population of sellers in *both* markets.

b $= b^{\text{in}} + b^{\text{fo}}$. Overall ratio of buyers to sellers in the whole economy.

B_f $:= b^{\text{fo}}/s^{\text{fo}}$. Ratio of buyers to sellers in formal market.

B_i $:= b^{\text{in}}/s^{\text{in}}$. Ratio of buyers to sellers in the informal market.

$c_{\text{se}}^{\text{in}}$ Cost of production in the informal market.

$c_{\text{se}}^{\text{fo}}$ Cost of production in the formal market. (Includes quality assurance, but not taxes or regulatory compliance.)

δ Discount factor (in section 4).

η Bargaining strength of informal sellers (in section 3).

g^{in} $:= v_{\text{bu}}^{\text{in}} - c_{\text{se}}^{\text{in}}$. The gains from trade in the informal market.

g^{fo} $:= v_{\text{bu}}^{\text{fo}} - c_{\text{se}}^{\text{fo}}$. The gains from trade in the formal market.

$L_{\text{bu}}^{\text{in}}$ Lump sum costs for informal buyers (e.g. inconvenience).

$L_{\text{bu}}^{\text{fo}}$ Lump sum costs for formal buyers (e.g. transportation and shoe leather costs).

$L_{\text{se}}^{\text{in}}$ Lump sum costs for informal sellers (e.g. crime risk, bribery, protection money, shoe leather).

$L_{\text{se}}^{\text{fo}}$ Lump sum costs for formal sellers (e.g. regulatory compliance, license fees, rent).

L_{bu} "Net" lump sum costs for formal buyers.

L_{se} "Net" lump sum costs for formal sellers.

$P_{\text{bu}}^{\text{in}}$ Match probability for informal buyers.

$P_{\text{bu}}^{\text{fo}}$ Match probability for formal buyers.

$P_{\text{se}}^{\text{in}}$ Match probability for informal sellers.

$P_{\text{se}}^{\text{fo}}$ Match probability for formal sellers.

q Expenditure on quality assurance technology by formal sellers.

$R_{\text{se}}^{\text{in}}$ $= 1 - T_{\text{se}}^{\text{in}}$, the residual earnings rate for informal sellers.

$R_{\text{se}}^{\text{fo}}$ $= 1 - T_{\text{se}}^{\text{fo}}$, the residual earnings rate for formal sellers.

R_{se} $= R_{\text{se}}^{\text{fo}}/R_{\text{se}}^{\text{in}}$, the "net" residual earnings rate for formal sellers.

s^{in} Proportion of sellers in the informal market.

s^{fo} Proportion of sellers in the formal market.

t Time (in dynamical interpretation of model).

$T_{\text{se}}^{\text{in}}$ Expected costs of monetary crime for informal sellers.

$T_{\text{se}}^{\text{fo}}$ Taxes and unit regulatory costs for formal sellers.

T_{ax} "Net" tax burden for formal sellers.

$u_{\text{bu}}^{\text{in}}$ Utility of a purchase for informal buyers.

$u_{\text{bu}}^{\text{fo}}$ Utility of a purchase for formal buyers.

$u_{\text{se}}^{\text{in}}$ Utility of a sale for informal sellers.

$u_{\text{se}}^{\text{fo}}$ Utility of a sale for formal sellers.

$U_{\text{bu}}^{\text{in}}$ $= P_{\text{bu}}^{\text{in}} u_{\text{bu}}^{\text{in}}$, the expected utility of informal buyers.

$U_{\text{bu}}^{\text{fo}}$ $= P_{\text{bu}}^{\text{fo}} u_{\text{bu}}^{\text{fo}}$, the expected utility of formal buyers.

$U_{\text{se}}^{\text{in}}$ $= P_{\text{se}}^{\text{in}} u_{\text{se}}^{\text{in}}$, the expected utility of informal sellers.

$U_{\text{se}}^{\text{fo}}$ $= P_{\text{se}}^{\text{fo}} u_{\text{se}}^{\text{fo}}$, the expected utility of formal sellers.

$v_{\text{bu}}^{\text{in}}$ Value of merchandise to informal buyer.

$v_{\text{bu}}^{\text{fo}}$ Value of merchandise to formal buyer.

Appendix D: Best response vector fields

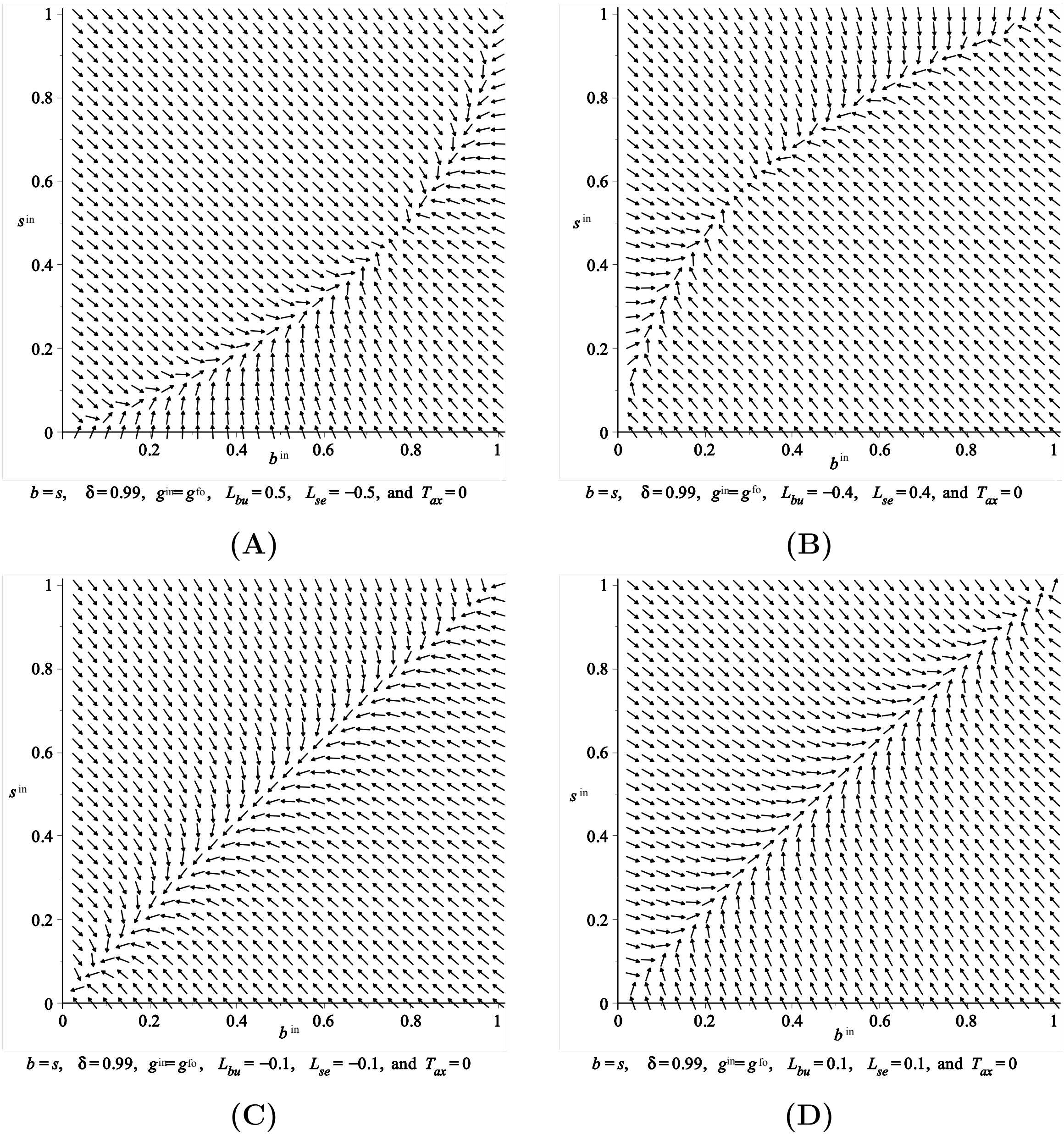

Figure 5: The vector fields generated by the best response differential equations (16), for the four cases shown in Figure 1. (A) **(A)** The mixed-market equilibrium defined by the crossing of $\mathcal{S}(0, -0.5)$ and $\mathcal{B}(0.5)$ is a global attractor. **(B)** The mixed-market equilibrium defined by the crossing of $\mathcal{S}(0, 0.4)$ and $\mathcal{B}(-0.4)$ is a global attractor. **(C)** The pure formal market equilibrium $(0, 0)$ is a global attractor. **(D)** The pure informal market equilibrium (b, s) is a global attractor.